ch-1 Measuring Instruments Dt-20-04-21

measurement :-

The result of comparision of unknown quantity w.r.t. predetermined standard.

(OR)

The conversion of physical quantity into numerical quantity and it must be represented along with corresponding unit.

→ unit respresents characteristics of the measurement.

Condition :-

1. Result must be represented in numerical form along with corresponding unit.
2. Instrument used must be acceptable by the user.
3. conversion from large to small quantity must be acceptable by the user.

Why :-

To wave the existance of practical world.

eg: Transformer

To design
To operate
To maintenance
To control
} we Require measurement.

Note:-

Measurement system is called as feed back control system → T/F

Methods of measurement

Direct methods :-

→ unknown quantity is compared directly.

→ Draw backs :-

* Human being errors may be more.

*. Practically not passible always practically not feasible always. Generally this method is not preferred.

Indirect method :-

-> In this method some instruments are used in which some effects are produced to give the unknown value

-> Generally it is preferred.

Types of Instrument :-

Mechanical Instrument :-

-> Large size
-> More weight
-> More power consumption
-> less portability
-> suitable only for stedy state applications.

Generally mechanical instruments are not used

Electrical Instrument :-

-> Size is less
-> weight is less
-> Power consumption is low
-> Speed of operation is high
-> Suitable for both steady state and dynamic appl.

Electronic Instrument :-

-> Size is least
-> Weight is least
-> Power consumption is least
-> Fastest speed of operation.
-> Suitable for both steady state and dynamic operation.

Electrical

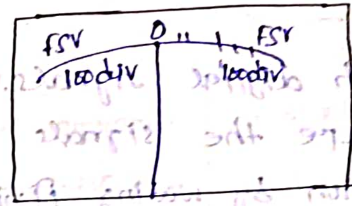
Null type | Deflection type.

Null type Instrument :-

-> To indicate zero value
eg. - Galvanometer

-> These are of centre zero scale instruments or bisided scale instruments.

eg.

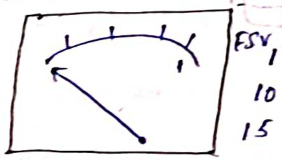


$$\text{each division} = \frac{.000000\ 1}{100} = .00000000\ 1$$

-> These are highly sensitive.

Deflection type Instrument :-

-> These are used to measure the unknown quantity by the deflection of pointer



Note :-

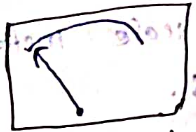
Though the null type instruments are more sensitive than deflection type instruments, deflection type instruments are prefered to measure the unknown quantity.

Analog instruments :-

-> These instruments works with analog signals.

-> Analog signals are the signal which varies conetact by waving infinite no of divisions.

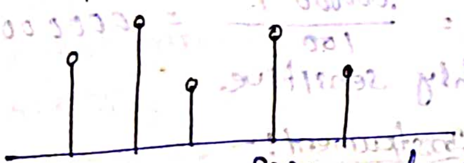
[continuous signal]



Digital instruments:-

→ These work with digital signals.

→ Digital signals are the signals which vary a distance step function by waring finite no. of division.



[Discreate signal]

ERROR ANALYSIS

Error:-

Deviation of measured value from the true value is called error.

$\epsilon \rightarrow$ Error

$A_t \rightarrow$ True value (actual value)

$A_m \rightarrow$ measured value

Error → $\boxed{\epsilon = A_m - A_t}$

$A_m - A_t = 0 \Rightarrow A_m = A_t$

$A_t - A_m = 0 \Rightarrow A_t = A_m$ → True value can be varied

$+A_m = +A_t$ → Direction is changed.

$\Rightarrow A_m = A_t$ ✗

* $A_m > A_t \Rightarrow \epsilon$ is +ve

$A_m < A_t \Rightarrow \epsilon$ is -ve

$\therefore \boxed{\epsilon \text{ is } \pm ve}$

Error → Static / Dynamic

Static	Dynamic
→ Absolute error	→ variable error
→ constant error	→ $e = A_m - A_t$
→ $\epsilon_0 = \delta A = A_m - A_t$	

Static error correction

→ It is denoted by δ_c

→ If δ_A is +ve, δ_c = -ve

δ_A is -ve, δ_c = +ve

$\therefore \boxed{\delta_c = -(\delta_A)} = -(A_m - A_t)$

$\boxed{\delta_c = A_t - A_m}$

Relative static error:-

→ It is denoted by ϵ_r

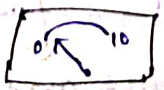
** It is expressed w.r.t A_t

$$\boxed{\% \, \epsilon_r = \frac{A_m - A_t}{A_t} \times 100}$$

→ It determines the quantity of the instrument.

→ It is also called as limiting error/uncertainty

eg.



let, $\epsilon_0 = \pm 0.1$

A_t	$\frac{A_m - A_t}{A_t}$
1	$\pm 0.1/1$
2	$\pm 0.1/2$
8	$\pm 0.1/8$
9	$\pm 0.1/9$

↓ varies (variable error)

→ It is a variable error

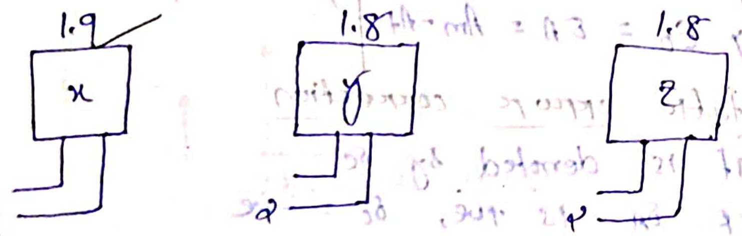
A) Static characteristics :-

These are the following static characteristics

1) Accuracy
2) Precision
3) Drift
4) Hysteresis
5) Linearity
6) Sensitivity
7) Resolution
8) Dead time and Dead zone

1) Accuracy :-

The closeness with which an instrument reading approaches the true value.



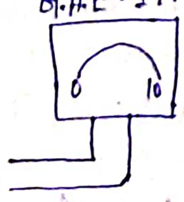
High accuracy

Guaranteed Accuracy Error → (GAE)

→ It is expressed w.r.t Full scale value (FSV)

→ It is a constant error.

G.A.E = ±1%



2
3
4
5
8

A_t	G.A.E
2	±0.1
3	±0.1
5	±0.1
8	±0.1

$\frac{\pm 1}{100} \times 10$ (FSV) $= 0.1$

constant

Q:- A 0-10 A ammeter has the GAE of ±1%. Find the limiting error when 2A, 5A & 10A of supply is given.

Sol^n Given, FSV = 10 A
GAE = ±1%

$\%$ Limiting error $= \frac{FSV}{A_t} \times \%\, GAE$

when, $A_t = 2A$

$\%\, L.E = \frac{10}{2} \times \pm 1\% = \pm 5\%$

$A_t = 5A$

$\%\, LE = \frac{10}{5} \times \pm 1\% = \pm 2\%$

$A_t = 10A$

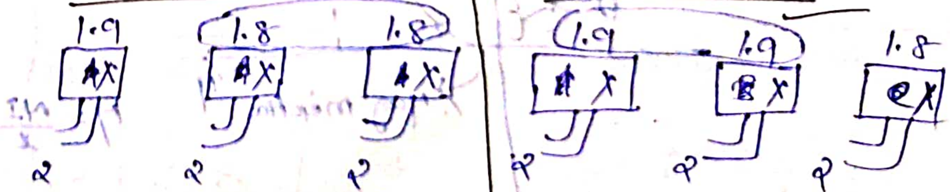
$\%\, LE = \frac{10}{10} \times \pm 1\% = \pm 1\%$

Note The limiting error is equal to the guaranteed Accuracy Error when true value (A_t) becomes Full scale value (FSV)

2) Precision :-

A prescribed value is either most repeated (or) most reproduced value.

Repeated | Reproduced



A A A | A B C

→ '1' observer | → 'n' observer

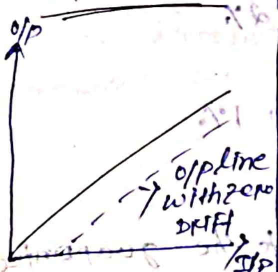
→ 'n' reading | → 'n' reading

* Highly accurate instrument may be precized → T/F
* precision confirms accuracy → T/F

3) Drift :—
→ It is the deviation.
→ used when characteristic curve are ploted.

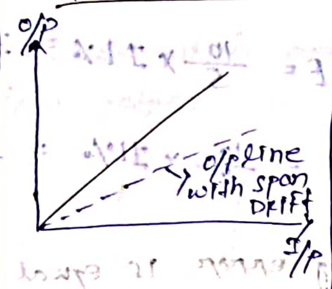
Note
If deviation is equal to zero, then the instrument is provided with no drift.

Zero Drift



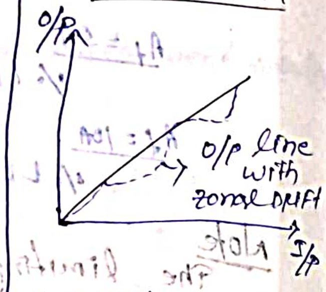
→ change in the drift is zero

Span Drift



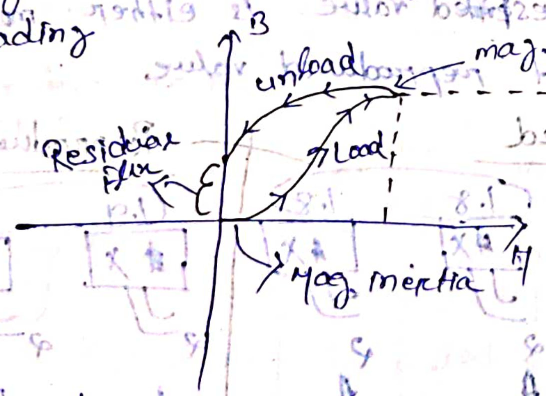
→ Drift is increased towards the fullspan of instrument

Zonal Drift



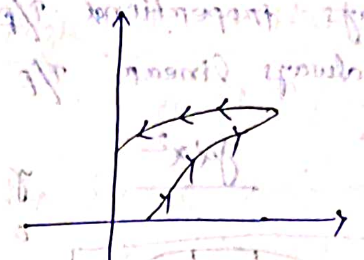
→ Drift is present in any zone.

4) Hysteresis :—
Exhibiting different property during loading and unloading

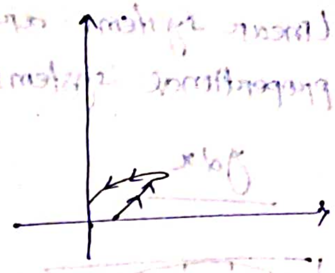


$H = \frac{NI}{l}$

Hysteresis is due to mag. Inertia
mag. saturation
Residual flux

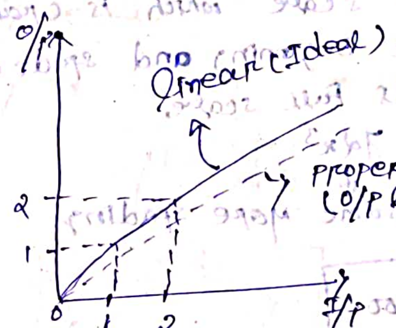


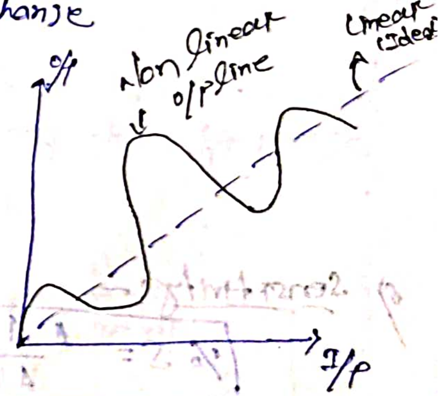
Hysteresis error is more because more deviation from loading to unloading



Hysteresis error is less because less deviation from loading to unloading

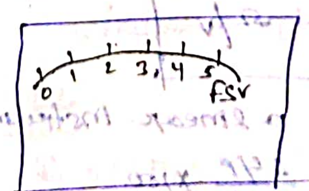
5) Linearity :—
A system is said to be linear when o/p change is equal to i/p change



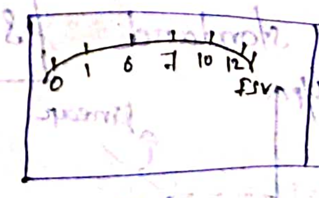


$$\% \text{ Nonlinearity} = \frac{\text{Max. deviation of o/p line from Ideal line}}{\text{Ad (or) Ideal line}} \times 100$$

Linear scale



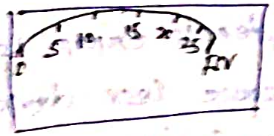
Nonlinear scale



Note
* Proportional system may be linear. T/F

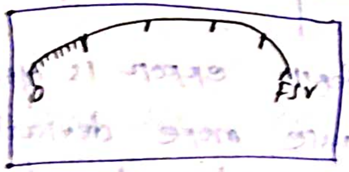
* Linear systems are always proportional T/F
* proportional systems are always linear T/F

* $y \propto x$



uniform / linear scale

$y \propto x^2$



y	x
0	0
1	1
4	2
9	3
16	4

nonlinear / Non uniform / Square law scale cramped at the beginning and spaced out at the FSR.

→ The scale which is cramped at the beginning and spaced out towards full scale.

$y \propto x^3$ → same more spacing

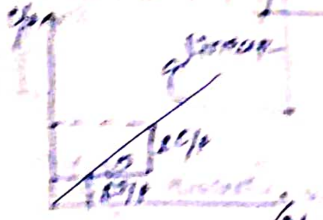
→ Sensitivity :-

$$\% S = \frac{\Delta O/P}{\Delta I/P} \times 100$$

Inverse sensitivity / deflection factor / scale factor

$$\% D = \frac{1}{S} = \frac{\Delta I/P}{\Delta O/P} \times 100$$

→ standard $S = \frac{1}{FSD}$ Ω/V



% S for a linear instrument

$$\% S = \frac{\Delta O/P}{\Delta I/P} \times 100$$

$= 1 \times 100$

$\boxed{\% S = 100\%}$ → linear instrument

Angle of sensitivity

$$\theta = \tan^{-1}\left(\frac{\Delta O/P}{\Delta I/P}\right)$$

* For linear Instrument

$\theta = \tan^{-1}(1)$

$\boxed{\theta = 45^\circ}$ → linear

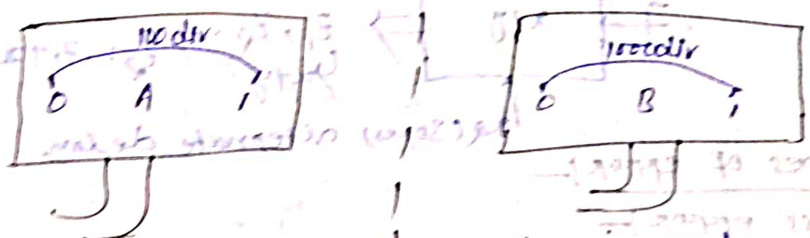
Note

* For a linear system single sensitivity is defined and the angle of sensitivity is 45° only
* For a nonlinear system multiple sensitivity can be defined.

→ Resolution :-

→ It is the smallest measurable I/P change

$$R = \frac{F.S.V}{\text{no. of divisions}}$$



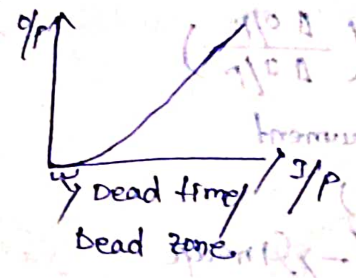
Each div = $\frac{1}{100}$ = 0.01 | Each div = $\frac{1}{1000}$ = 0.001

Instrument 'B' has good Resolution

Note

Resolution is improved with no. of division increased.

8) **Dead time and Dead zone:-**



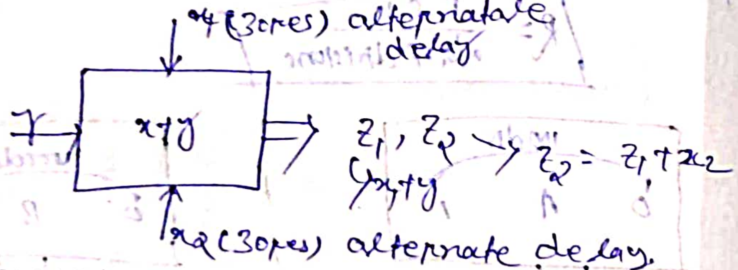
Note:-
Generally it is defined at origin.

B) **Dynamic charecteristics:-**

1) **Speed of response:-**
The rapidly of an instrument to respond.

* We prefer as high as possible speed of responce.

2) **Measuring delay:-**
It is also called as delay



$z_1, z_2 \rightarrow z_2 = z_1 + x_2$

Types of error:-

Gross error:-
These are the human being mistake in observing the reading, recording the value and calculating the required parameters

Systematic error:-

a) Instrumental ⇒ (i) Repeated loading and unload
(ii) Ensure of instrument

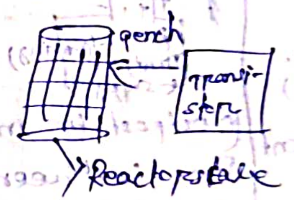
b) Environmental → Environmental change.

c) observational → paralax error.

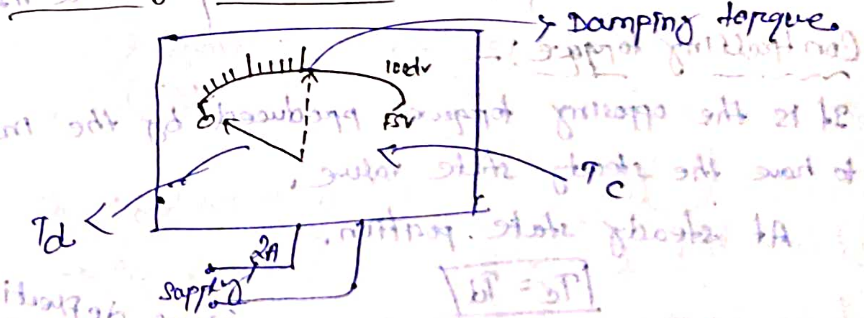
Random error:-
The error which can't be predicted (no cause in found)

Basic Instruments

Indicating	Recording	Integrating
Eg- A, V, W	Eg- seismo graph	Eg-> i) energymeter $E = \int P dt$ = w sec (wattmeter) ii) $Q = \int I dt$ = A-h meter

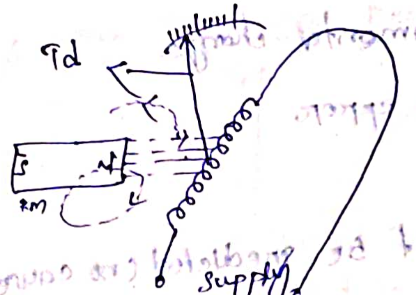


Indicating Instruments:-

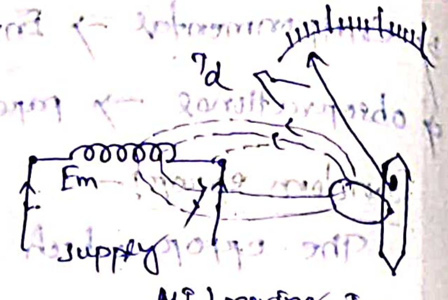


Deflecting Torque:-
-> It is the torque produced in the instrument i.e required to move the pointer from the intial position (zero position)

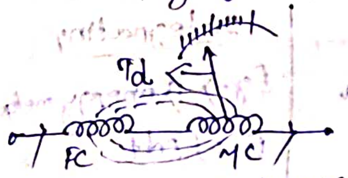
Effects
-> Magnetic effect
-> Electromagnetic effect
-> Induction effect
-> Electrostatic effect.



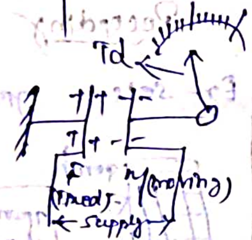
PMMC → permanent mag. moving coil.
[Force betⁿ PM and current carrying coil]



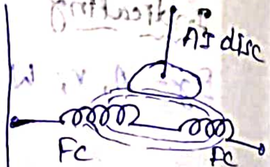
(MI) moving Iron.
(Force between current carrying coils & Iron rod)



(EMMC) (electromag. moving coil)
(Force betⁿ two current carrying coil)



(electrostatic inst.)
Force between two plate



Induction type
Force betⁿ two current carrying coil and Al disc

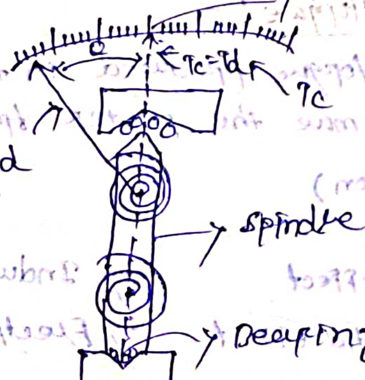
Controlling torque :–

→ It is the opposing torque produced by the instrument to have the steady state value.

At steady state position.

$T_c = T_d$

Spring control :–



$T_c \propto \theta$

$T_c = K_c\theta$ where K_c = spring constant.

If $T_d \propto I$

At steady state,

$T_c = T_d$

$\theta \propto T_d$
$T_c \propto T_d$
So, $T_c \propto \theta$

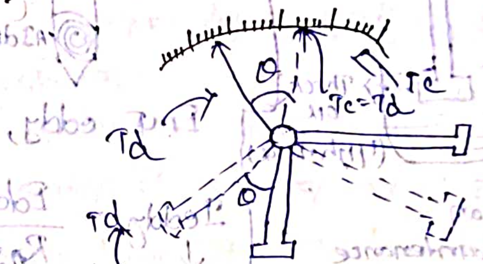
$\theta \propto I$ → linear/uniform scale

Advantages :–

→ uniform / linear scale.

→ It can be mounted either horizontally / vertically

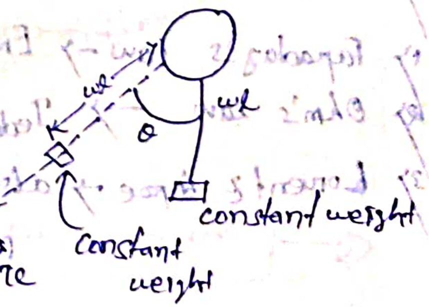
Gravity control :–



→ Gravity control weight force (T_c)

So, $T_c \propto W l \sin\theta$

⇒ $T_c \propto \sin\theta$



If $T_d \propto I$

At steady state position

$T_c = T_d$

$I \propto \sin\theta$ → Nonlinear / Non-uniform scale

Disadvantages :–

→ Non linear / non uniform scale.

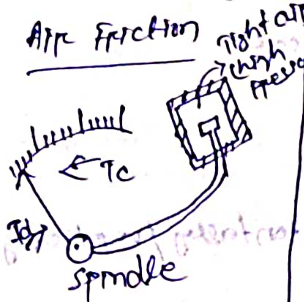
→ It should be mounted vertically only.

Note
Spring control method is generally prefered

Damping torque:-

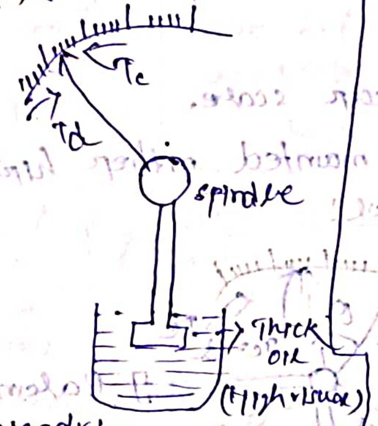
Defn - This is the torque produced in instruments to stop the oscillation of the pointer about the it's final steady state position.

Air friction



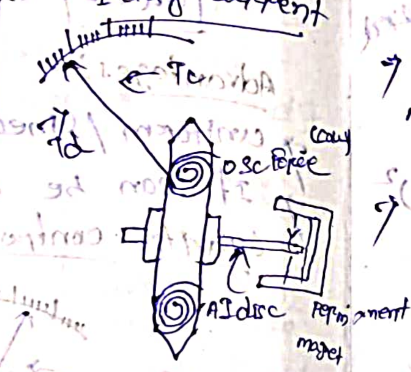
adv → i) simple
ii) clean
iii) less maintenance
iv) can be horizontal & vertically place

Fluid friction



Disadv:-
i) not clean
ii) more maintenance
iii) only vertically place.

Eddy current



Emf eddy,

$I_{eddy} = \frac{E_{eddy}}{R_{AI}}$
↓
Icirculating.

→ So, due to which a force is generated (final force) (result)

1) Faraday's Law → EMF eddy
2) Ohm's law → I_{eddy}
3) Lorentz force → a force (resulting force) (Current carrying core place in mag. field)
4) Lenz law → The generated force opposes the oscillating force which is causing it, so finally

→ As the pointer makes oscillation about it's final steady state position, the AI disc gets oscillated. Therefore flux will be cut by the AI disc, then emf will be induced (Faraday's law of electro-mag. induction). This emf is called as eddy emf.

⇒ $I_{eddy} = \frac{E_{eddy}}{R_{AI}}$ i.e current is produced on the AI disc (ohm's law)

→ Now the current carrying AI disc placed in the magnetic field experiences mechanical force (Lorentz's force eqn)

→ This resultant force opposes the main cause (oscillating force) (Lenz's law).

There by oscillation of pointer is stopped.

Conclusion:-

Order of effectiveness → Eddy > Fluid > Air
Order of preference → Eddy > Air > Fluid
} Imp.

② Moving Iron (M I) Instruments:-

The most common ammeters and voltmeters are of moving iron type.

Torque equation of M.I Instruments:-

T_d → Deflecting torque
$d\theta$ → Small deflection.

| Mechanical work done in dt sec = $T_d \cdot d\theta$ | → ① |

let, I → Initial current
L → Instrument inductance

If the current increases by dI, then the deflection changes by $d\theta$ and inductance by dL.

The voltage, $v = \frac{d}{dt}(LI)$

$= L\frac{dI}{dt} + I\frac{dL}{dt}$

$v\,dt = L\,dI + I\,dL$

$vI\,dt = LI\,dI + I^2 dL$ → (2)

($v \cdot I \cdot dt$ = P · t = energy) → Energy supplied in dt sec.

Energy stored in 'I' sec $= \frac{1}{2} LI^2$

Energy stored in 't+dt' sec $= \frac{1}{2}(L+dL)(I+dI)^2$

So, energy stored in dt sec,

$= \frac{1}{2}(L+dL)(I+dI)^2 - \frac{1}{2}LI^2$

$= \frac{1}{2}(L+dL)(I^2+2I\,dI+d^2I^2) - \frac{1}{2}LI^2$

Neglecting the second and higher order terms,

Energy stored in dt sec $= LI\,dI + \frac{1}{2} I^2 dL$ → (3)

From the conservation of energy,

Electrical energy supplied = Increase in stored energy + mechanical work done

$\Rightarrow LI\,dI + I^2 dL = LI\,dI + \frac{1}{2} I^2 dL + T_d\,d\theta$

$\Rightarrow \frac{1}{2} I^2 dL = T_d\,d\theta$

$\Rightarrow T_d = \frac{1}{2}\frac{I^2 dL}{d\theta}$

$\Rightarrow T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$ → (4)

As $dL \propto d\theta$

$\frac{dL}{d\theta}$ = constant

So, $T_d \propto I^2$

Here, T in N-m
I in Ampere
L in Henry
θ in radian

Controlling torque :-

The moving system is provided by control spring and deflecting torque T_d is balanced by controlling torque T_c.

$T_c = k_c\,\theta$

where, k_c = control spring constant N/m/rad.

θ = Deflection, rad.

At the final steady state,

deflecting torque (T_d) = control torque (T_c)

$\Rightarrow T_c = T_d$

$\Rightarrow k_c\,\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$

$\Rightarrow \theta = \frac{\frac{1}{2} I^2 \frac{dL}{d\theta}}{k_c}$

Deflection $\theta = \frac{1}{2}\frac{I^2}{k_c}\frac{dL}{d\theta}$ → (5)

i.e Deflection is proportional to square of rms value of operating current.

$\theta \propto I^2$ → Nonlinear / Non uniform scale.

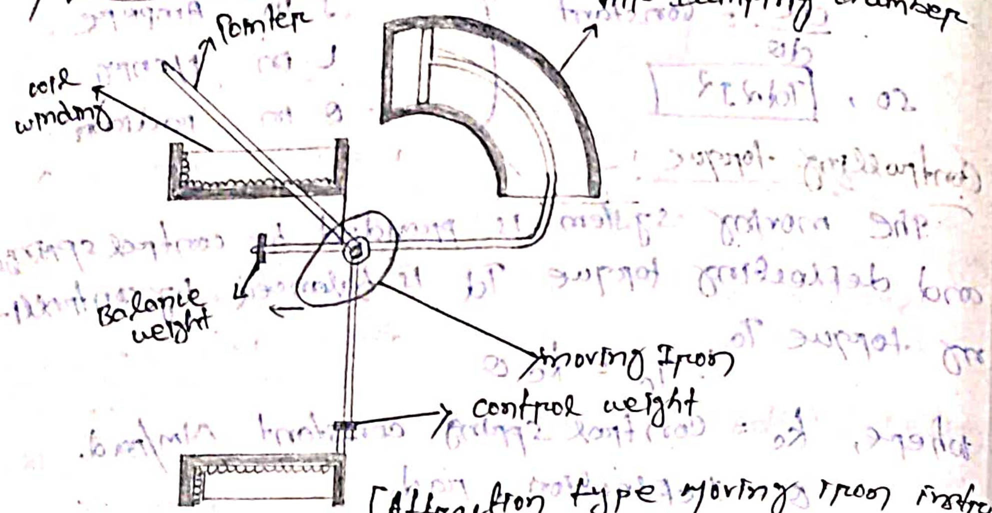
Classification of moving iron instruments :-

Moving iron instruments are of two types:-

i) Attraction type.

ii) Repulsion type.

i) Attraction type :-



(Attraction type moving iron instrument)

The above Figure shows an attraction type moving iron instrument.

- The coil is Flat and has a narrow slot like opening.
- The moving iron is a Flat disc or sector.
- When the current Flows through the coil a magnetic Field is produced and moving iron moves from the weaker Field outside the coil to the stronger Field inside it i.e attracted inword.
- The controlling torque is provided by gravity control method which is vertically moveed.
- Damping is provided by air Freiction with the help of a light aluminium position (attached to moving system) which moves in a Fixed chamber closed at one end at as shown in the Figure.

ii) Repulsion type :-

In this case, there are two vanes inside the coil, one is Fixed and other movable.

- When current Flows through the coil, there is a repulsion between two vanes resulting in the movement of moving vane.

Two different vane design are used;

a) Radial vane type
b) Co-axial vane type

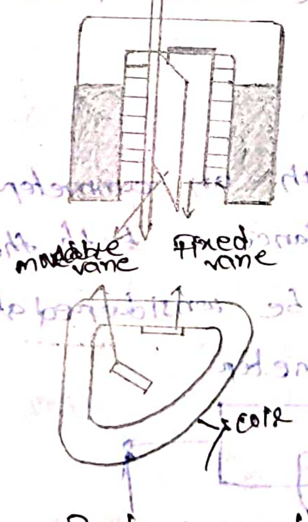
a) Radial vane type

In this type vanes are radial strip of iron.

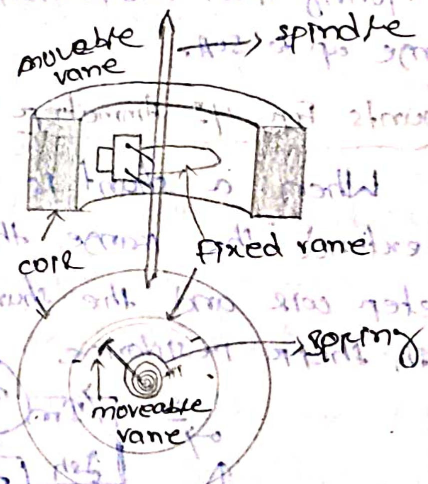
The Fixed vane is attached to the coil and the moveable one to the spindle of instrument.

b) co-axial vane type :-

In this type of instruments the Fixed or moving vales are sections of co-axial cylinder.



a) Radial vane type



(B) co-axial vane type

→ The controlling torque is provided by spring.

→ The damping torque is produced by air friction in attraction type instrument.

→ The operating magnetic field in MI instrument is very weak so eddy current damping is not used.

Note :-

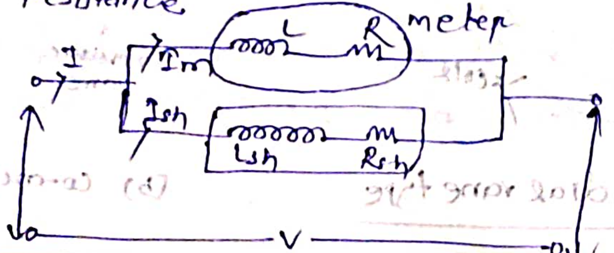
Irrespective of direction of current in the coil of the instrument, the iron vanes are so magnetised that there is always a force of attraction in attraction type and repulsion in the repulsion type instruments. So MI instruments are unpolarised (or) unidirectional i.e they are independent of the direction in which the current passes. So MI instrument can be used for both ac and dc.

Extension of range of moving iron instrument :-

Moving iron instruments can be built for a range up to 50A.

Shunts for MI Ammeter :-

When a shunt is used with an ammeter to extend the range the inductance of both the meter coil and the shunt must be considered along with their resistance.



The currents in the meter and shunt are in inverse ratio of their impedance

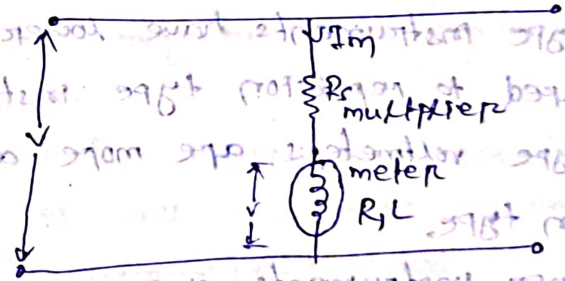
$$\frac{I_{sh}}{I_m} = \sqrt{\frac{R^2+(\omega L)^2}{(R_{sh})^2+(\omega L_{sh})^2}}$$

$$= \frac{R_{sh}}{R}\sqrt{\frac{1+\left(\frac{\omega L}{R}\right)^2}{1+\left(\frac{\omega L_{sh}}{R_{sh}}\right)^2}}$$

→ The ratio of two currents depends on frequency.

Multipliers for MI voltmeters :-

The voltage range of MI instruments can be extended by the use of a series resistance / multiplier with the meter.



Let, R = Resistance of meter

L = Inductance of meter

I_m = Meter current for full scale deflection

ω = angular frequency

The voltage drop across the meter for full scale deflection

$$V = I_m\sqrt{R^2+\omega^2L^2}$$

Let, 'V' is the voltage to be measured.

R_s → Resistance of non-inductive multiplier.

Total resistance of ckt = $R + R_s$

Total inductance of ckt = L

Total impedance of ckt = $\sqrt{(R+R_s)^2 + \omega^2 L^2}$

The current in the meter, $I_m = \dfrac{V}{\sqrt{(R+R_s)^2 + \omega^2 L^2}}$

$\Rightarrow V = I_m \sqrt{(R+R_s)^2 + \omega^2 L^2}$

Then the voltage multiplying power

$$m = \frac{V}{v} = \frac{\sqrt{(R+R_s)^2 + \omega^2 L^2}}{\sqrt{R^2 + \omega^2 L^2}}$$

Comparision between Attraction and Repulsion type Instruments :-

→ Attraction type instruments have lower inductance as compared to repulsion type instruments.

→ Attraction type voltmeters are more accurate than repulsion type.

→ Repulsion type instruments are more suitable for economical production and nearly uniform scale.

Errors in Moving Iron Instruments :-

There are two types of errors which occur in MI instruments.

1) Errors which occur with both ac and dc.

2) Errors which occur only with ac.

1) Errors with both DC and AC :-

a) Hysteresis Error :-

This error occurs as the value of flux density is different for the same current for ascending and descending values.

→ This error can be minimized by making the iron parts small so that they demagnetise themselves quickly (or) by making the iron parts at low flux density value.

b) Temperature Error :-

The temperature error in MI instruments arises due to temperature co-efficient of spring.

→ The error may be 0.02% per °C change in temperature.

→ In voltmeter, errors are caused due to self-heating of coil and series resistance. So the series resistance (R_s) should be made up of ~~Magna~~ Manganin with small temperature coefficient.

→ The value of series resistance (R_s) should be very large as compared to coil resistance (R) to minimize the errors due to self heating.

7) stray magnetic field:-

The errors due to stray magnetic fields i.e fields other than the operating magnetic field can be minimized by using an iron case (or) a thin shield over working parts.

2) Errors with AC only:-

1) Frequency errors:-

changes in the frequency may cause errors due to change of reactance of working coil and eddy current setup in the metal parts of instrument.

-> Reactance of instrument coil:-

In case of voltmeter where an additional R_s is used in series with instrument coil, errors due to reactance occurs.

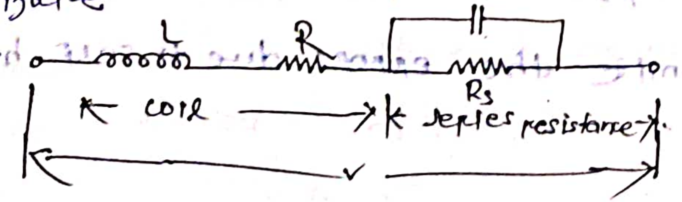
let, R, L -> resistance and inductance of instrument

R_s -> series resistance

so, total current $I = \frac{V}{\sqrt{(R+R_s)^2 + \omega^2 L^2}}$

-> As in MI instrument, deflection 'θ' depends on the current through coil, the deflection at high frequency is less than low frequency

-> This type of error can be reduced by using a capacitor 'C' across the series resistance R_s as shown in figure



Eddy current:-

These errors are caused by the eddy currents induced in the iron parts of the instruments.

-> At low frequency, eddy current error increases and at high frequency this error is practically constant. Due to this MI instruments are not suitable for the frequencies above 125 Hz.

Advantages and Disadvantages of MI instruments

1) Universal use:-

These instruments can be used for both ac & dc

-> As the hysteresis effect causes large error in measurement of d.c it is rarely used for d.c application.

-> As hysteresis causes no problems for ac, MI instruments are widely used in industry for measurement of ac voltage and current.

2) Less friction error:-

As the torque-weight ratio is quite high, the frictional errors are quite small.

3) cheapness:-

As single type of moving element could cover the entire range to be measured, MI instruments are very cheap as compared to other instruments.

4) Robustness:-

Due to the simple construction and no current carrying moving parts, the instruments are robust.

5) **Accuracy:-**
These instruments are very accurate and are used within the limits of industrial grades.

6) **Scale:-**
MI instruments are available with 240° circular scale.
-> The scale of MI instruments is not uniform and cramped at lower end, so accurate readings are not possible.

7) **Errors:-**
These instruments suffer from errors due to hysteresis, frequency change and stray magnetic field.

8) **Wave form errors:-**
The non-linearity of B-H curve of iron result in the waveform error as the deflecting torque is not exactly proportional to the square of current.

9) **Difference bet^n ac and dc calibration:-**
There is a difference bet^n ac and dc calibration due to the effect of inductance and eddy current when the meter is used for ac.

Problem The inductance of a moving iron instrument is given by, $L = (10 + 5\theta - \theta^2)\mu H$, where θ is the deflection in radian from zero position. The spring constant is 12×10^{-6} N/rad. Estimate the deflection for a current of 5A.

Sol^n Given that, $L = (10 + 5\theta - \theta^2)\mu H$

Spring constant, $K_c = 12 \times 10^{-6}$ N/rad

current, $I = 5A$

$\theta = ?$

As we know deflection, $\theta = \frac{1}{2}\frac{I^2}{K_c}\frac{dL}{d\theta}$ —— (1)

$\frac{dL}{d\theta} = (10 + 5\theta - \theta^2)\mu H$

$= (0 + 5 - 2\theta)\mu H/rad$

$= 5 - 2\theta \,\mu H/rad$

$= 5 - 2\theta \times 10^{-6}$ H/rad

putting the value in eq^n (1) we get,

$\theta = \frac{1}{2}\frac{(5)^2}{12 \times 10^{-6}} \times 5 - 2\theta \times 10^{-6}$

$= \frac{25}{24}(5 - 2\theta)$

$\Rightarrow 24\theta = 125 - 50\theta$

$\Rightarrow 24\theta + 50\theta = 125$

$\Rightarrow \theta = \frac{125}{74} = 1.689$ rad.

Prob The inductance of a moving iron ammeter with a full scale of 90° at 1.5A, is given by the expression $L = (200 + 40\theta - 4\theta^2 - \theta^3)\mu H$, where θ is the deflection in radian from zero position. Estimate the angular deflection of the pointer for a current of 1.0 A.

Given that, $L = 200 + 40\theta - 4\theta^2 - \theta^3 \,\mu H$

for, $I = 1.5A$, $\theta = 90° = \pi/2$ rad

As, $\theta = \frac{1}{2}\frac{I^2}{K_c}\frac{dL}{d\theta}$ —— (1)

$\frac{dL}{d\theta} = 200 + 40\theta - 4\theta^2 - \theta^3$

$= (40 - 8\theta - 3\theta^2)\mu H/rad$

for, $\theta = \pi/2$ rad, $\frac{dL}{d\theta} = 40 - \frac{8 \times \pi}{2} - 3\left(\frac{\pi}{2}\right)^2$

$= 20.03 \simeq 20 \mu H/rad$

putting the value of $\frac{dL}{d\theta}$ in eqn (1)

$$\frac{\pi}{2} = \frac{1}{2}\frac{(1.5)^2}{K_c} \times 20 \times 10^{-6}$$

$$\Rightarrow K_c = \frac{(1.5)^2 \times 20 \times 10^{-6}}{\pi}$$

$$= 14.33 \times 10^{-6} \text{ Nm/rad}$$

Now, for I = 1 A.

$$\theta = \frac{1}{2}\frac{I^2}{K_c}\frac{dL}{d\theta}$$

$$= \frac{1}{2} \times \frac{(1)^2}{14.33 \times 10^{-6}} \times 40 - 8\theta - 3\theta^2 \times 10^{-6}$$

$$\Rightarrow 28.66\theta = 40 - 8\theta - 3\theta^2$$

$$\Rightarrow 28.66\theta + 8\theta + 3\theta^2 = 40$$

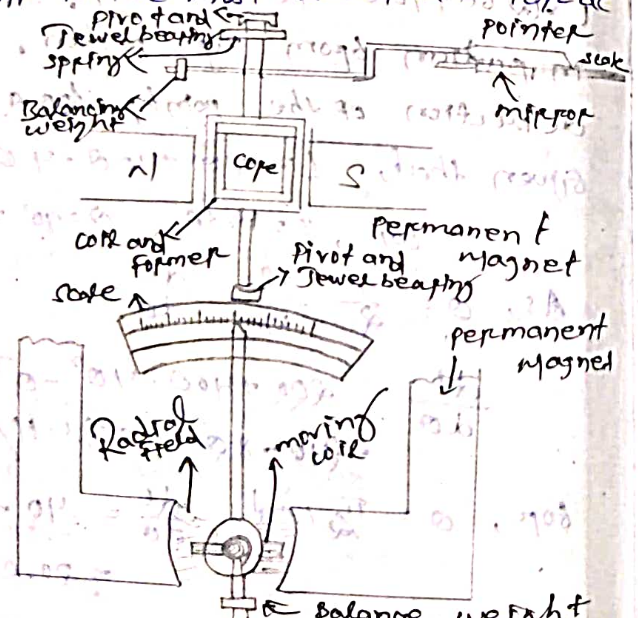
$$\Rightarrow 3\theta^2 + 36.66\theta - 40 = 0$$

$$\Rightarrow \boxed{\theta = 1.008 \text{ rad}} \text{ or } -13.22 \text{ rad}$$

Permanent magnet moving coil Instrument :-

PMMC instrument is the most accurate for dc measurement.

Construction :-



(PMMC Instrument)

Moving coil :-

→ The moving coil is wound with many turns of enamelled (or) silk covered copper wire.

→ The coil is mounted on a rectangular aluminium former which is pivoted on jewel bearings.

→ The coils move freely in the field of permanent magnet.

→ Voltmeters coils are wound on metal frames and ammeter coils are wound on non-magnetic former.

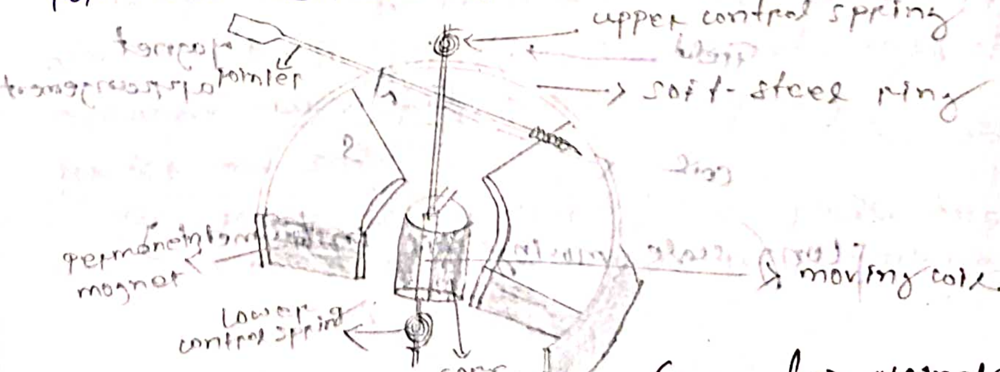
Magnet systems :-

Due to the development of materials like Alcomax and Alnico having high coercive force, smaller magnet length is used in PMMC instruments.

→ The flux density in PMMC instruments various from 0.1 wb/m² to 1 wb/m²

Concentric magnet :-

→ To obtain longer movement of the pointer and longer angular swing of the coil, concentric magnet assembly is used. It produces radial flux which extends over 250 or more. This type of construction is used for panel type instrument and some portable instruments.



(Concentric magnet assembly)

Air cored coil:-

→ In air cored coil, scale length is form 120° to 240° or even 300°.

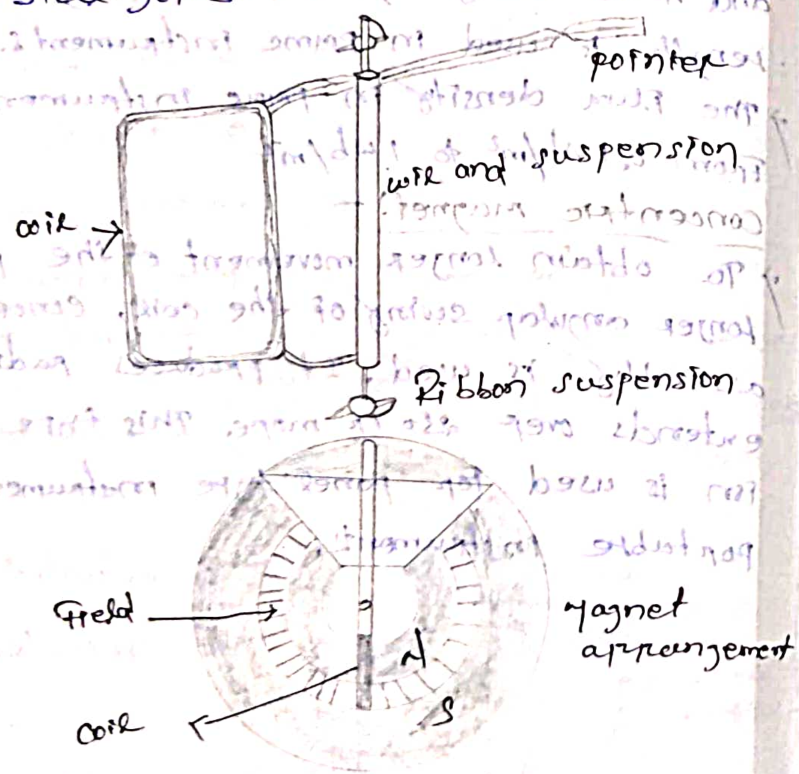
→ It gives better resolution of reading for same scale range.

Core magnet system:-

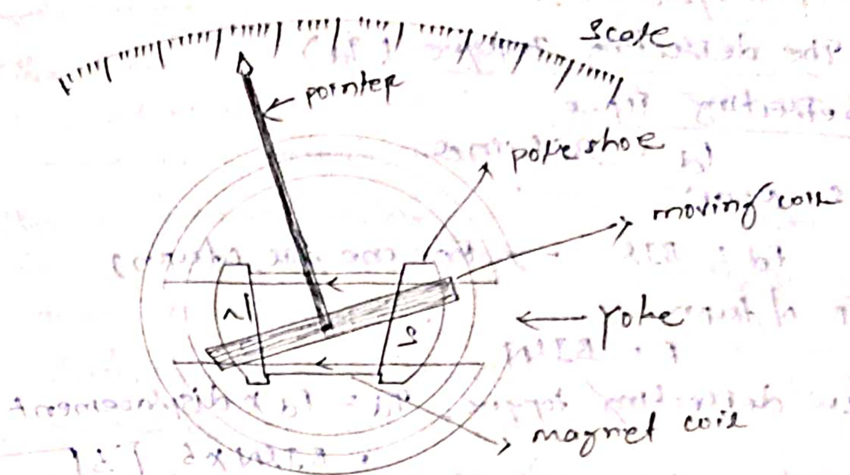
Due to the development of ALNICO, the same magnet itself behaves as core.

The moving coil moves over the magnet.

→ The active sides of the moving coil are located in the uniform radial field bet? pole pieces and the steel yoke.



[Long scale moving coil instrument]



[Core magnet construction]

Advantages:-

→ It is relatively unaffected by the external magnetic field.

→ It eliminates the magnetic shunting effects.

→ It eliminates the magnetic shielding in the form of iron case.

Control torque:-

When the coil is supported between two jewel bearings the control torque is provided by two phosphor bronze hair springs.

Damping Torque:-

Damping torque is produced by movement of aluminium former moving in the magnetic field of permanent magnet (Eddy current Damping)

Pointer and scale:-

The pointer is carried by the spindle and moves over a graduated scale is of light-weight

Torque Equation :-

The deflecting Torque (T_d)

Deflecting Force

$F_d = BIl\sin\theta$

As, $\theta = 90°$

$F_d = BIl \rightarrow$ For one coil (turn)

For N turn,

$F = BIlN$

Now deflecting torque $T_d = F_d \times$ displacement

$= BIlN \times b$ [b, l]

where, $l \rightarrow$ length of rectangular coil

$b \rightarrow$ width of the coil turn

So, $T_d = BIlNb$

$\boxed{T_d = BINA \ N\text{-}m}$ [As $A = l \times b$]

where, B = Flux density wb/m^2

I = Current

N = No. of turns and A = Area of coil

As, $T_d = BINA$ N-m.

B,N, A are constants.

So, $\boxed{T_d \propto I}$

Controlling Torque :-

The spring control provides the restoring Torque, $T_c = k_c\theta$

For Final steady state deflection,

$T_c = T_d$

$\Rightarrow k_c\theta = BINA$

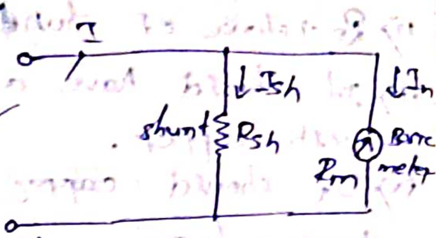
$\Rightarrow \boxed{\theta = \frac{BINA}{k_c}}$ Again B,N,A, k_c = constant

$\boxed{\theta \propto I}$

Ammeter shunts :-

When heavy currents are to be measured, the major part of the current is bypassed through a low resistance called shunt.

The resistance of shunt can be calculated by following ckt. analysis.



R_m = Internal resistance of coil (Ω)

R_{sh} = resistance of shunt (Ω)

$I_m = I_{FS}$ = Full scale deflection current (A).

I_{sh} = shunt current (A)

I = current to be measurement (A)

As shunt resistance is in parallel with meter, voltage drop of shunt = voltage drop of meter.

$\Rightarrow I_{sh}R_{sh} = I_m R_m$

$\Rightarrow R_{sh} = \frac{I_m R_m}{I_{sh}}$

Again, $I_{sh} = I - I_m$

So, $R_{sh} = \frac{I_m R_m}{(I - I_m)}$

$\Rightarrow \frac{I - I_m}{I_m} = \frac{R_m}{R_{sh}}$

$\Rightarrow \frac{I}{I_m} - 1 = \frac{R_m}{R_{sh}}$

$\Rightarrow \frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}$

As, $\frac{I}{I_m} = m$ = multiplying power of shunt $= \frac{\text{total current}}{\text{meter current}}$

$\Rightarrow m = 1 + \frac{R_m}{R_{sh}}$

$\Rightarrow \frac{R_m}{R_{sh}} = m - 1 \Rightarrow \boxed{R_{sh} = \frac{R_m}{m-1}}$

General requirements of shunt :-

i) Temperature co-efficient of shunt and instrument should be low and same.

ii) Resistance of shunt should not vary with time.

iii) It should have a low thermal electromotive force with copper.

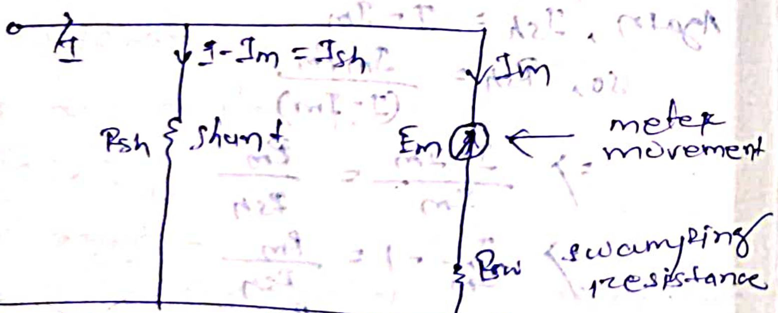
iv) It should carry current without excess rise of temperature.

Note :-

* manganin is used for shunt of d.c instrument.
* Constantan is used for shunt of ac instrument.

Effect of temperature changes in ammeter :-

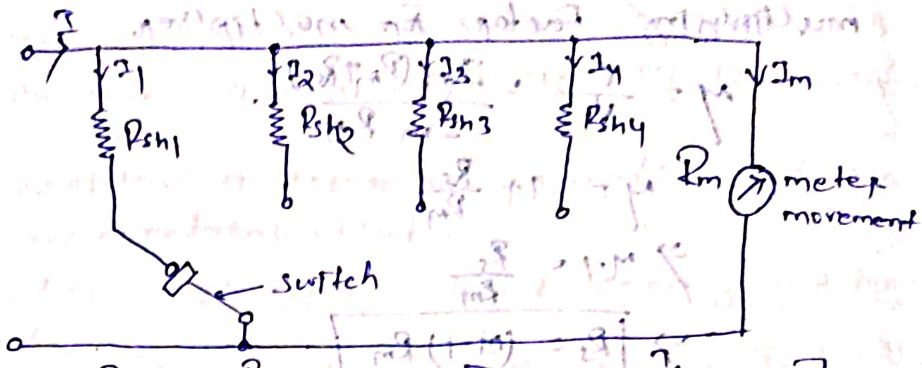
To compensate the effect of tempr change, a "swamping resistance" of manganin having a resistance 20-30 times of coil resistance is connected in series with the coil and a shunt of manganin is connected across the combination.



(meter shunt with swamping resistance)

Multirange Ammeter :-

→ The current range of a d.c ammeter can be extended by a number of shunts selected by a range switch. Such ammeter is called multirange ammeter.



$$R_{sh_1} = \frac{R_m}{m_1 - 1} \qquad m_1 = \frac{I_1}{I_m}$$

$$R_{sh_2} = \frac{R_m}{m_2 - 1} \qquad m_2 = \frac{I_2}{I_m}$$

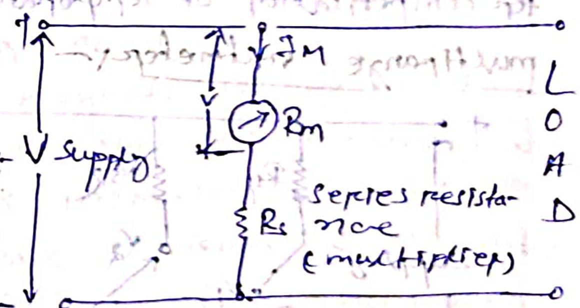
$$R_{sh_3} = \frac{R_m}{m_3 - 1} \qquad m_3 = \frac{I_3}{I_m}$$

$$R_{sh_4} = \frac{R_m}{m_4 - 1} \qquad m_4 = \frac{I_4}{I_m}$$

Voltmeter multipliers :-

A series resistance connected across the meter to limit the current in voltmeter is known as multiplier.

The value of multiplier is calculated as below.



$I_m = I_{fs}$ = full scale deflection current

R_m = meter resistance

R_s = multiplier resistance

v = voltage across the meter

V = Full range voltage of instrument.

From the ckt,

$$v = I_m R_m$$

$$V = I_m (R_s + R_m)$$

$$\therefore R_s = \frac{V - I_m R_m}{I_m} = \frac{V}{I_m} - R_m$$

multiplying factor for multiplier

$$m = \frac{V}{V_1} = \frac{I_m (R_m + R_s)}{I_m R_m}$$

$$m = 1 + \frac{R_s}{R_m}$$

$$\Rightarrow m - 1 = \frac{R_s}{R_m}$$

$$\Rightarrow R_s = (m-1) R_m$$

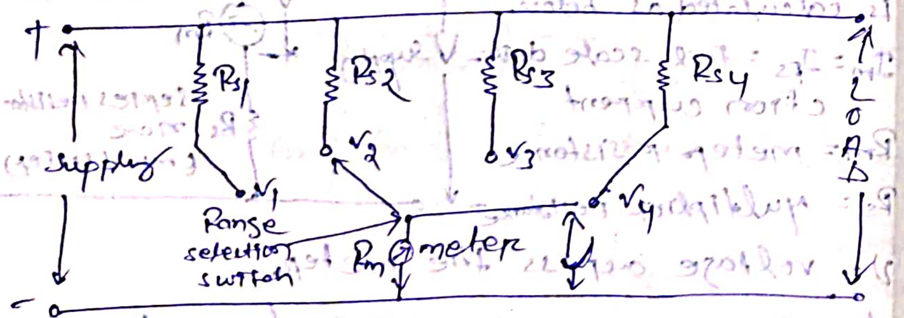
General requirement of multiplier:–

- Resistance should not change with time.
- Resistance change with temp^r should be small.
- should be non-inductively wound for ac motor.

Effect of temp^r change in voltmeter:–

multiplier resistance is made up of manganin having a negligible resistance temp. coefficient for compensation of temperature change.

multirange voltmeter:–



The multirange voltmeter can be constructed by connecting resistance in series with meter.

$R_{s1} = (m_1 - 1) R_m$, $R_{s2} = (m_2 - 1) R_m$

$R_{s3} = (m_3 - 1) R_m$, $R_{s4} = (m_4 - 1) R_m$

where, $m_1 = \frac{v_1}{v}$, $m_2 = \frac{v_2}{v}$, $m_3 = \frac{v_3}{v}$, $m_4 = \frac{v_4}{v}$

Errors in PMMC Instruments:–

The main sources of errors in PMMC instruments are:–

i) weakening of permanent magnets due to ageing at temperatures effects.

ii) weakening of springs due to ageing and temp^r affects.

iii) Change of resistance of the moving coil with temperature.

magnets:–

Magnets are aged by heat and vibration treatment which results in the loss of initial magnetism.

springs:–

The weakening of springs with time can be reduced by careful use of material and pre-ageing during manufacture.

- The weakening of magnets decreases the deflection for a particular value of current but weakening of springs increases the deflection.
- In PMMC 1°c increase of temperature reduces the strength of spring by about 0.04 percent and reduces flux density.

moving coil:–

- The indication of PMMC instrument for a constant current would decrease by 0.04 per °c rise in temperature.
- But incase of PMMC voltmeter, due to the large series resistance of negligible temporally coefficient eliminates the error due to temperature.

Advantages and disadvantages of PMMC instrument

Advantages:—

- The scale is uniformly devided.
- How power consumption ie 25μW to 200μW.
- Torque-weight ratio is high which gives a high accuracy.
- A single instrument can be used for many different current and voltage ranges by different values of shunts and multipliers.
- The large flux density we high as 0.5 wb/m² reduce the errors due to stray magnetic field.
- Self-shielding magnets in case of core magnet construction is useful in aircraft and aerospace application.

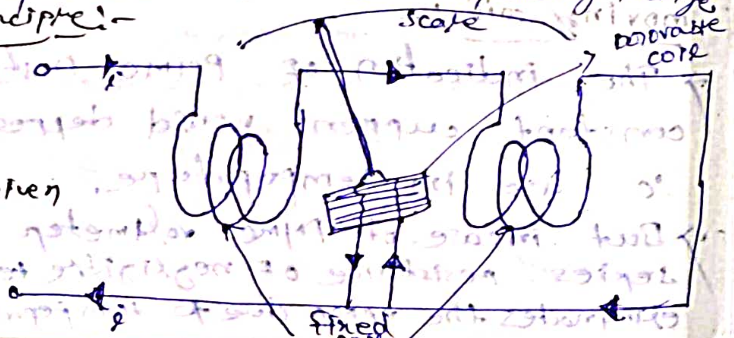
Disadvantages:—

- These instruments are useful for only d.c as when current reverses the torque reverses.
- The cost of these instrument is high than moving iron instruments.

Dynamometer Type Instruments:—

Electrodynamometer type instruments are used as ac voltmeters and ammeters both in the range of power frequencies and audio frequency range.

Operating Principle:—



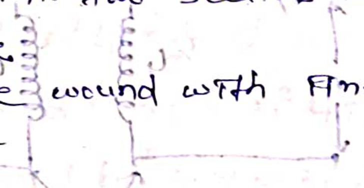
(Draw diagram from previous given photo).

- In case of PMMC, torque is in one direction during one half cycle and opposite direction during the other half cycle. so the pointer would swing back and fourth around the zero point.
- If the direction of flux reverses each time current through the movable coil reverses, then unidirectional torque would be produced for both +ve and -ve half cycle.

Construction:—

Fixed coil:—

- The field is produced by a fixed coil.
- The coil is divided into two sections to give uniform field near the centre.



- The fixed coils are wound with fine wires supported by ceramic

Moving coil:—

- The moving coil is wound either as a self-sustaining coil or else as a non-metalic former.
- A metalic former can not be used as it would produce eddy current by the alternating field.

Control:—

The controlling torque is provided by two control springs.

Moving system:—

- The moving coil is mound on a aluminium spindle
- The moving system also carries the counter weights and pointer.

Damping:-

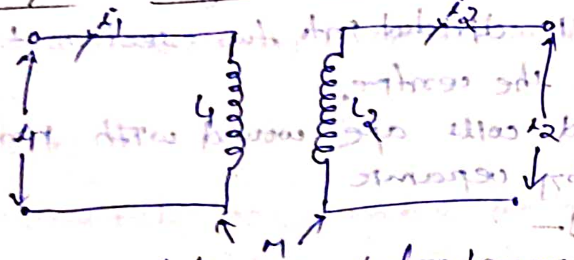
Air friction damping is used in this instrument.

Damping is provided by a pair of aluminium vanes attached to the spindle at the bottom.

Shielding:-

As the operating field produced by the fixed coil is nearly 0.005 to 0.006 wb/m² which is very weak field, it is necessary to shield electrodynamometer type instruments from the external stray magnetic field.

Torque eqⁿ of Electrodynamometer Instruments:-



(Ckt representation of electrodynamometer Instrument)

Let, i_1 = Instantaneous value of current in fixed coil; A.

i_2 = Instantaneous value of current in moving coil; A

L_1 = self inductance of fixed coil, H.

L_2 = self inductance of moving coil, H,

M = Mutual inductance betⁿ fixed coil and moving coil H.

Flux linkage of coil-1, $\phi_1 = L_1 i_1 + M i_2$

Flux linkage of coil-2, $\phi_2 = L_2 i_2 + M i_1$

→ Electrical I/P energy $= e_1 i_1 dt + e_2 i_2 dt$

As, $e_1 = \frac{d\phi_1}{dt}$ and $e_2 = \frac{d\phi_2}{dt}$

$\Rightarrow e_1 dt = d\phi_1$ and $e_2 dt = d\phi_2$

So, electrical I/P energy $= (e_1 dt) i_1 + (e_2 dt) i_2$

$= d\phi_1 i_1 + d\phi_2 i_2$ —— ①

Now, $d\phi_1 = d(L_1 i_1 + M i_2)$

$= L_1 di_1 + i_1 dL + M di_2 + i_2 dm$

$d\phi_2 = d(L_2 i_2 + M i_1)$

$= L_2 di_2 + i_2 dL_2 + M di_1 + i_1 dm$

putting the value of $d\phi_1$ and $d\phi_2$ in equation ①

Electrical input energy =

$i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dm + i_1 M di_2 + i_2 L_2 di_2 + i_2^2 dL_2 + i_1 i_2 dm + i_2 M di_1$ —— ②

Energy stored in magnetic field.

$= \frac{1}{2} i_1^2 L_1 + \frac{1}{2} i_2^2 L_2 + i_1 i_2 M$

So change in energy stored $= d(\frac{1}{2} i_1^2 L_1 + \frac{1}{2} i_2^2 L_2 + i_1 i_2 M)$

$= \frac{1}{2} d(i_1^2) L_1 + \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} d(i_2^2) L_2 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dm + i_1 M di_2 + i_2 M di_1$

$= \frac{1}{2}(2i_1) di_1 L_1 + \frac{1}{2} i_1^2 dL_1 + \frac{1}{2}(2i_2) di_2 L_2 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dm + i_1 M di_2 + i_2 M di_1$

$= i_1 L_1 di_1 + \frac{1}{2} i_1^2 dL_1 + i_2 L_2 di_2 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dm + i_1 M di_2 + i_2 M di_1$ —— ③

According to principle of conservation of energy

Total electrical input energy = change in energy stored + mechanical energy

⇒ Mechanical energy = Total electrical energy − change in energy stored

Subtracting eqⁿ ③ from eqⁿ ② we get.

Mechanical energy $= i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dm + i_1 m di_2 + i_2 L_2 di_2 + i_2^2 dL_2 + i_1 i_2 dm + i_2 m di_1 - (i_1 L_1 di_1 + \frac{i_1^2}{2} dL_1 + i_2 L_2 di_2 + \frac{i_2^2}{2} dL_2 + i_1 m di_2 + i_2 m di_1 + i_1 i_2 dm)$

$= i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dm + i_1 m di_2 + i_2 L_2 di_2 + i_2^2 dL_2 + i_1 i_2 dm + i_2 m di_1 - i_1 L_1 di_1 - \frac{i_1^2}{2} dL_1 - i_2 L_2 di_2 - \frac{i_2^2}{2} dL_2 - i_1 m di_2 - i_2 m di_1 - i_1 i_2 dm$

$= i_1^2 dL_1 - \frac{i_1^2}{2} dL_1 + i_2^2 dL_2 - \frac{i_2^2}{2} dL_2 + i_1 i_2 dm$

$= \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dm$

As L_1 and L_2 are constant $dL_1 = 0$ & $dL_2 = 0$

So mechanical energy $= i_1 i_2 dm$ —— (4)

⇒ Let, T_i = Instantaneous deflecting torque

$d\theta$ = change in deflection

So, mechanical energy = workdone $= T_i\, d\theta$ —— (5)

So equating eqn (4) & eqn (5) we get,

$T_i\, d\theta = i_1 i_2 dm$

$$\boxed{T_i = i_1 i_2 \frac{dm}{d\theta}} \quad \text{—— (6)}$$

The average deflection of torque over a complete cycle

$$T_d = \frac{1}{T}\int_0^T T_i\, dt$$

$$= \frac{1}{T}\int_0^T i_1 i_2 \frac{dm}{d\theta}\, dt$$

Let, $i_1 = I_{m_1} \sin\omega t$

$i_2 = I_{m_2} \sin(\omega t - \phi)$

where, ϕ = phase difference (phase angle) bet^n i_1 & i_2

Then, $T_d = \frac{1}{2\pi}\int_0^{2\pi} I_{m_1}\sin\omega t \cdot I_{m_2}\sin(\omega t - \phi) \cdot \frac{dm}{d\theta} \cdot d(\omega t)$

$$= \frac{dm}{d\theta} \frac{(I_{m_1})(I_{m_2})}{2\pi} \int_0^{2\pi} \sin\omega t \sin(\omega t - \phi)\, d(\omega t)$$

Note

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

$\cos(A-B) - \cos(A+B) = \cos A \cdot \cos B + \sin A \cdot \sin B - \cos A \cdot \cos B + \sin A \cdot \sin B$

$= 2 \sin A \cdot \sin B$

$\Rightarrow \sin A \cdot \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

Similarly, $\sin\omega t \cdot \sin(\omega t - \phi)$

$= \frac{1}{2}[\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)]$

$= \frac{1}{2}[\cos\phi - \cos(2\omega t - \phi)]$

Putting the value of $\sin\omega t \cdot \sin(\omega t - \phi)$

$\Rightarrow T_d = \frac{dm}{d\theta} \cdot \frac{I_{m_1} I_{m_2}}{2\pi} \cdot \frac{1}{2} \int_0^{2\pi} [\cos\phi - \cos(2\omega t - \phi)]\, d(\omega t)$

$= \frac{dm}{d\theta} \cdot \frac{I_{m_1} I_{m_2}}{2\pi \times 2} \left[\cos\phi \int_0^{2\pi} d(\omega t) - \int_0^{2\pi} \cos(2\omega t - \phi)\, d\omega t\right]$

$= \frac{dm}{d\theta} \cdot \frac{I_{m_1} I_{m_2}}{2\pi \times 2} \left[\cos\phi\,(2\pi - 0) - \frac{\sin(2\omega t - \phi)}{2\omega t}\Big|_0^{2\pi}\right]$

$= \frac{dm}{d\theta} \cdot \frac{I_{m_1} I_{m_2}}{2\pi \times 2} \cdot \cos\phi \cdot (2\pi)$

$T_d = \frac{I_{m_1} I_{m_2}}{2} \cos\phi \frac{dm}{d\theta}$

As $I_1 = \frac{I_{m_1}}{\sqrt{2}}$ & $I_2 = \frac{I_{m_2}}{\sqrt{2}}$

$$\boxed{T_d = I_1 I_2 \cos\phi \frac{dm}{d\theta}} \quad \text{—— (8)}$$

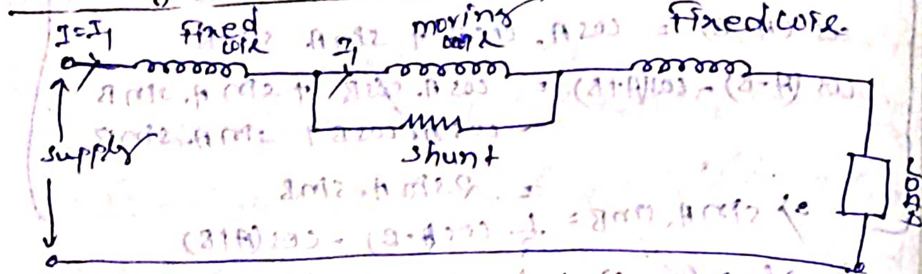
where, I_1 & I_2 are rms value of current flowing through coils.

⇒ At steady state position $T_d = T_c$

$\Rightarrow I_1 I_2 \cos\phi \frac{dm}{d\theta} = K_c \theta$

$$\Rightarrow \boxed{\text{Deflection } \theta = \frac{I_1 I_2}{K_c} \cos\phi \frac{dm}{d\theta}} \quad \text{—— (9)}$$

Electrodynamometer Ammeters:—



In this case fixed coil and moving coils are connected in series.

so, $I_1 = I_2 = I$ and $\phi = 0$

so deflecting torque $T_d = I^2 \cos 0 \, \frac{dm}{d\theta}$

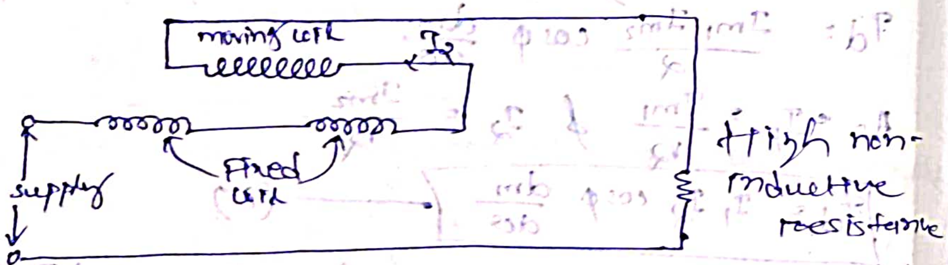
$$\boxed{T_d = I^2 \frac{dm}{d\theta}}$$

$$\boxed{\text{Deflection } \theta = \frac{I^2}{k_c} \frac{dm}{d\theta}}$$

→ The current through the moving coil should not exceed 100 mA.

Electrodynamometer voltmeters:—

Deflecting Torque $= I_1 I_2 \cos\phi \, \frac{dm}{d\theta}$



In this case $I_1 = I_2 = V/Z$, $\phi = 0$

$$T_d = \left(\frac{V}{Z}\right)\left(\frac{V}{Z}\right) \cdot \cos 0 \, \frac{dm}{d\theta}$$

$$\boxed{T_d = \frac{V^2}{Z^2} \frac{dm}{d\theta}}$$

where, V = voltage across the instrument.
Z = Impedance of the instrument ckt

$$\boxed{\text{Deflection, } \theta = \frac{V^2}{K_c Z^2} \, \frac{dm}{d\theta}}$$

→ Electrodynamometer, voltmeter range is 10 to 30 V.

Errors in electrodynamometer instruments:—

There are various errors in dynamometer type of instruments.

i) low torque/weigh ratio
ii) Frequency
iii) Eddy currents
iv) External magnetic fields
v) Temperature change

i) Torque/weigh ratio:—
As the flux linkage is very small the deflecting torque is low.

→ A low torque/weight ratio indicates a heavy moving system, so the frictional losses is high, so frictional errors are high.

ii) Frequency:—
The frequency error of dynamometer type instrument is very large due to the variation of self-reactance of coils with frequency in case of voltmeter.

Deflection $\theta = \left(\frac{V^2}{KZ^2}\right) \frac{dm}{d\theta}$

Impedance, $Z = \sqrt{R^2 + (2\pi f L)^2}$

iii) Eddy current:—
The effect of eddy currents is to produce a torque betn moving coil and adjacent metal part.

→ The eddy current produces frequency error.

iv) **External magnetic fields:—**

As the operating field is very weak, the external magnetic field interference is high, so shielding is required to avoid stray magnetic field.

v) **Temperature changes:—**

Self heating of coils produces errors.

→ To reduce the temperature errors, compensating resistors are used.

Advantages and Disadvantages of electrodynamometer instruments:—

Advantages:—

→ As all the coils are air-cored, these instrument are free from hysteresis and eddy current errors.

→ These instruments can be used up to frequency range of 40–500 Hz.

→ These instruments can be used on both a.c and dc.

→ Electrodynamometer voltmeters are most accurate type of ac voltmeter.

Disadvantages:—

→ As the torque/weight ratio is the low, the sensitivity is also low.

→ Low torque/weight ratio gives increased frictional losses.

→ More expensive as compared to PMMC & MI.

→ Sensitive to overload and mechanical impact.

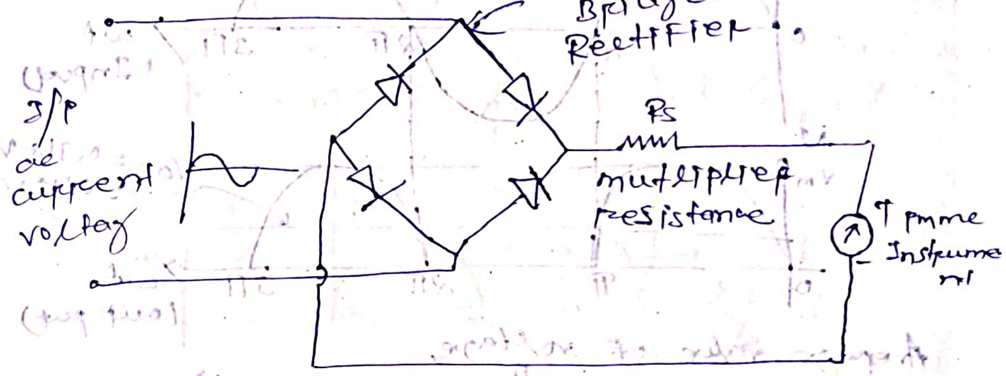
→ The operating magnetic field is weak but the large operating current results in large power consumption than PMMC

→ Non-uniform scale. General frequency range is 0–125 Hz

Rectifier Type Instruments:—

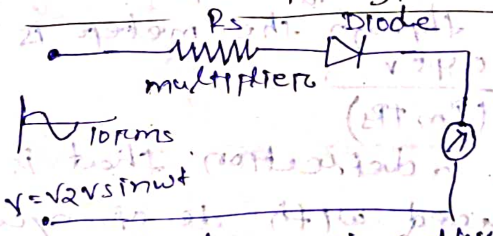
→ Rectifier type instruments are used for measurement of ac voltages and currents by using a rectifier element which converts ac to dc and then by using a dc meter (PMMC) to indicate the value of rectified ac.

→ These instruments are primarily used as voltmeter.



(Basic arrangement of Rectifier instrument using a full wave rectifier ckt)

Halfwave Rectifier type Instrument:—



[Voltmeter using HWR ckt]

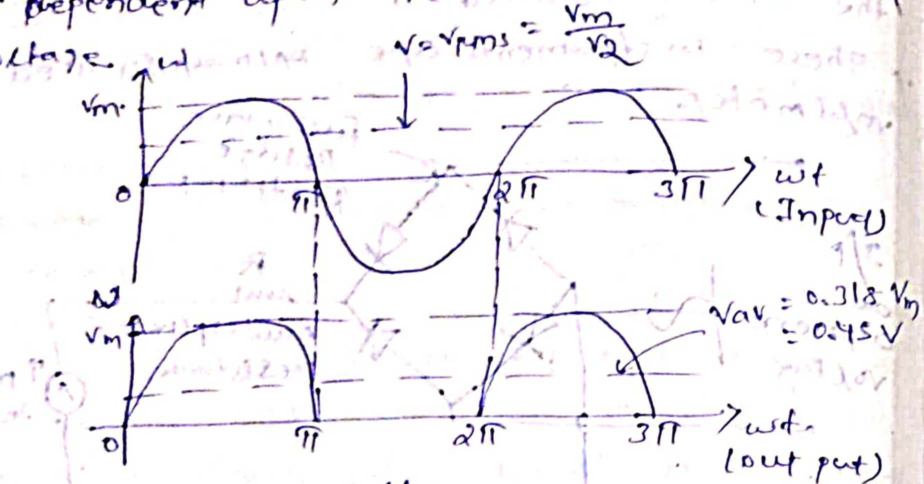
When dc voltage $V_{dc} = V$ is applied to ckt, the meter current $I_m = \frac{V}{R_S + R_m}$

→ When ac voltage $v = V_m \sin\omega t = \sqrt{2}\, V \sin\omega t$ is applied (where V_m = Max/peak ac voltage, V = V_{rms} voltage of ac

$V_{rms} = \frac{V_{max}}{\sqrt{2}} \Rightarrow V_m = \sqrt{2}\, V_{rms} = \sqrt{2}\, V$)

→ The rectified unidirectional pulsating voltage produces a pulsating current and hence a pulsating torque.

→ The PMMC indicates the deflection corresponding to the average value of current which is dependent upon the average value of applied voltage.



Average value of voltage,

$$V_{av} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t \, d(\omega t)$$

$$= \frac{V_m}{\pi} = 0.318 V_m = 0.318 \times \sqrt{2} V$$

$$V_{av} = 0.45 V$$

So, the current through the meter is

$$I_{av} = \frac{V_{av}}{R_m + R_s} = \frac{0.45 V}{(R_m + R_s)}$$

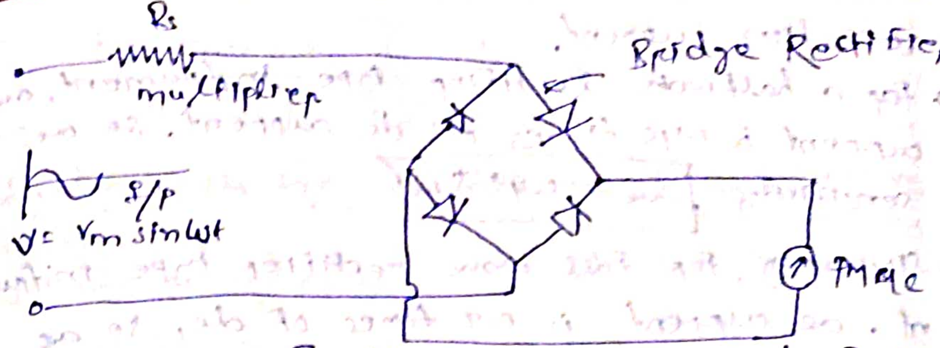
i.e it produces a deflection that is 0.45 times that produced with dc of equal magnitude V.

<u>Full wave Rectifier type Instrument :-</u>

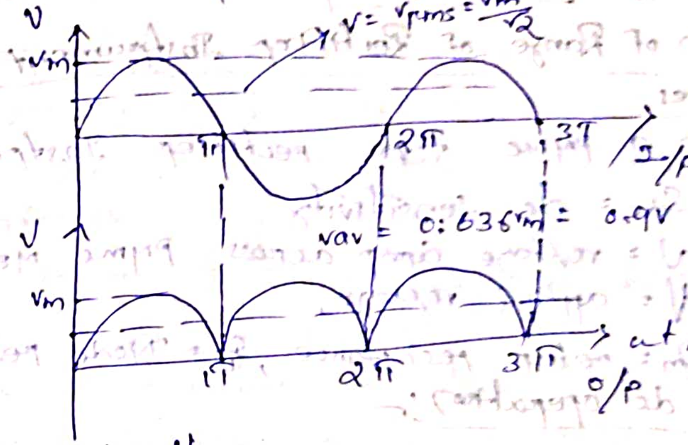
→ When a dc voltage $V_{dc} = V$ is applied to the rectifier the current through the meter, $I_m = \frac{V}{R_m + R_s}$

→ When a ac voltage $v = V_m \sin\omega t = \sqrt{2} V \sin\omega t$

where, $V = V_{rms} = \frac{V_m}{\sqrt{2}} \Rightarrow V_m = \sqrt{2} V$



[voltmeter using F.W Bridge Rectifier.



is applied, then average voltage appearing across the meter

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d(\omega t)$$

$$= \frac{2V_m}{\pi} = 0.636 V_m = 0.9V$$

So, the average current in the meter

$$I_{av} = \frac{0.9V}{R_m + R_s}$$

There fore the deflection with ac is 0.9 times that produced with dc for same value of voltage V.

<u>Sensitivity of Rectifier type Instruments :-</u>

The dc sensitivity of rectifier type instrument $S_{dc} = \frac{1}{I_{fs}}$ V/Ω where I_{fs} is full scale deflection current.

- for a halfwave Rectifier type instrument, ac current is 0.45 times of dc current, so ac sensitivity $S_{ac} = 0.45\, S_{dc}$
- Similarly, for full wave rectifier type instrument, ac current is 0.9 times of dc, so ac sensitivity $S_{ac} = 0.9\, S_{dc}$

Extension of Range of Rectifier Instrument as voltmetres.

let in a PMMC type rectifier instrument

S_{dc} = dc sensitivity

v = voltage drop across PMMC meter

V = applied voltage

R_m = meter resistance, R_d = Diode resistance

so for dc operation :-

The series multiplier resistance for Half wave Rectifier

$$R_s = \left(\frac{V}{I_{fs}}\right) - R_m - R_d$$

$$= \frac{1}{I_{fs}} \cdot V - R_m - R_d$$

$$R_s = S_{dc} \cdot V - R_m - R_d$$

similarly R_s for full wave Rectifier

$$R_s = \frac{V}{I_{fs}} - R_m - 2R_d$$

$$R_s = S_{dc} \cdot V - R_m - 2R_d$$

for ac operation :-

$R_s = S_{ac} \cdot V - R_m - R_d$ (Half wave)

$$R_s = 0.45\, S_{dc} V - R_m - R_d$$

For full wave Rectifier

$$R_s = 0.9\, S_{dc} V - R_m - R_d$$

Factors affecting performance of Rectifier type instruments

Disadvantages / errors :-

- Effect of rectified wave form.
- Rectifier resistance.
- Temperature changes.
- Rectifier capacitance
- Decrease in sensitivity.

Advantages:-

- The frequency range extends from about 20 Hz to high audio frequency.
- These instruments have lower operating current for voltmeter than other ac instrument.
- They have practically uniform scale for most ranges.
- Their accuracy is about 5%.

loading effect :-

The sensitivity of rectifier type ac voltmeter is less than dc voltmeter. so loading effect of ac rectifier voltmeter is more than of dc voltmeter.

Prob-1 A permanent magnet moving coil instrument has a coil of dimensions 15mm × 12mm. The flux density in the air gap is 1.8×10^{-3} wb/m² and the spring constant is 0.14×10^{-6} Nm/rad. Determine the no. of turns required to produce an angular deflection of 90 degree when a current of 5mA flowing through the coil.

Sol^n Given that,

$A = 15mm \times 12mm$

$= 15\times10^{-3}m \times 12\times10^{-3}m$

$B = 1.8\times10^{-3}$ wb/m²

$K_c = 0.14\times10^{-6}$ Nm/rad.

$\theta = 90° = \pi/2$ rad.

$I = 5mA = 5\times10^{-3}A$

Note:
$180° = \pi$ radian
$1° = \frac{\pi}{180}$ radian
$90° = \frac{\pi}{180}\times 90 = \frac{\pi}{2}$ radian

At final steady state,

$T_d = T_c$

$\Rightarrow BINA = K_c\theta$

$\Rightarrow N = \frac{K_c\theta}{BIA}$

$\Rightarrow N = \frac{0.14\times10^{-6}\times\pi/2}{1.8\times10^{-3}\times5\times10^{-3}\times15\times10^{-3}\times12\times10^{-3}}$

$\Rightarrow \boxed{N = 136}$

> ∴ Multiplying power, $m = I/I_m$
> $= 1 + \frac{R_m}{R_{sh}}$
> ∴ Resistance of shunt, $R_{sh} = \frac{R_m}{(m-1)}$

Prob-2 Find the multiplying power of a shunt of 200 Ω resistance used with a galvanometer of 1000 Ω resistance. Determine the value of shunt resistance to give a multiplying power of 50.

Sol^n Given that,

$R_{sh} = 200\Omega$, $R_m = 1000\Omega$ — Find m = ?

Find R_{sh} = ? for $m = 50$

→ multiplying power $m = 1 + \frac{R_m}{R_{sh}}$

$= 1 + \frac{1000}{200}$ (= 1 + 5)

$\boxed{m = 6}$

→ when, $m = 50$

shunt resistance, $R_{sh} = \frac{R_m}{m-1}$

$= \frac{1000}{50-1} = \frac{1000}{49}$

$R_{sh} = 20.4\Omega$

Prob-3 Design a multi-range d.c multi-ammeter using a basic movement with an internal resistance $R_m = 50\Omega$ and a full scale deflection current $I_m = 1mA$. The ranges required are 0-10mA; 0-50mA; 0-100mA; and 0-500mA.

Sol^n (i) 0-10mA range: multiplying power.

$m = I/I_m = \frac{10}{1} = 10$

Resistance of shunt, $R_{sh_1} = \frac{R_m}{m-1} = \frac{50}{10-1} = 5.55\Omega$

(ii) 0-50mA range, $m = 50/1 = 50$

∴ $R_{sh_2} = \frac{50}{50-1} = 1.03\Omega$

(iii) 0-100mA range, $m = \frac{100}{1} = 100$

∴ $R_{sh_3} = \frac{50}{100-1} = 0.506\Omega$

(iv) 0-500mA range, $m = \frac{500}{1} = 500$

∴ $R_{sh_4} = \frac{50}{500-1} = 0.1\Omega$

Prob-4 A moving coil instrument gives a full scale deflection of 10 mA when the potential difference across its terminals is 100 mV. Calculate,

i) The shunt resistance for a full scale deflection corresponding to 100 A.

ii) The series resistance for full scale reading with 1000 V.

Calculate the power dissipation in each case.

Sol^n Given, meter current $I_m = 10\,mA$

$V_m = 100\,mV$,

meter resistance, $R_m = \frac{V_m}{I_m} = \frac{100\,mV}{10\,mA} = \boxed{R_m = 10\,\Omega}$

i) Shunt multiplying factor $m = \frac{I}{I_m}$

$m = \frac{100}{10\times10^{-3}} = \frac{100000}{10}$

$\boxed{m = 10000}$

shunt resistance $R_{sh} = \frac{R_m}{m-1}$

$= \frac{10}{10000-1} = 1\times10^{-3} = 0.001\,\Omega$

power dissipation $= V_m I = 100\times10^{-3}\times100 = 10\,W$.

ii) voltage multiplying factor, $m = \frac{V}{V_m} = \frac{1000}{100\times10^{-3}}$

$\boxed{m = 10{,}000}$

series multiplier, $R_s = (m-1)R_m$

$= (10{,}000-1)\,10 \simeq 100000\,\Omega$

$\simeq 100\,k\Omega$

power dissipation $= V I_m = (1000)\times(10\times10^{-3})$

$= 10\,W$.

Prob-5 which meter has a greater sensitivity? Meter A having a range of 0-10 V and a multiplier resistance of 18 kΩ or meter B with a range of 0-300 V and a multiplier resistance of 298 kΩ. Both the meter movements have a resistance of 2 kΩ.

Sol^n Given that, $R_m = 2\,k\Omega$, $R_s = 18\,k\Omega$

For meter A (0-10V)

Total resistance = Meter resistance + multiplier resistance

$R = R_m + R_s$

$R = 2\,k\Omega + 18\,k\Omega$

$R = 20\,k\Omega$

sensitivity of meter, A $= \frac{1}{I_m} = \frac{1}{V/R}$

$= \frac{R}{V} = \frac{20}{10} = 2\,k\Omega/V$

For meter B (0-300V)

Total resistance, $R = R_m + R_s$

$= 2\,k\Omega + 298\,k\Omega$

$R = 300\,k\Omega$

sensitivity of meter, B $= \frac{1}{I_m} = \frac{1}{V/R} = R/V = \frac{300}{300}$

$= 1\,k\Omega$

Prob-6 The inductance of a 25 A electrodynamic ammeter changes uniformly at the rate of 0.0035 µH/degree. The spring constant is 10^{-6} N-m/degree. Determine angular deflection at full scale.

Sol^n Final steady deflection $\theta = \frac{I^2}{k} \cdot \frac{dm}{d\theta}$

Spring constant 'k' is given in N-m/degree and therefore, we must express $(dm/d\theta)$ in H/rad if deflection is to be found in degrees.

$\frac{dm}{d\theta} = \frac{0.0035}{\pi/180}\times10^{-6} = 0.2\times10^{-6}$ N-m/rad.

∴ Deflection, $\theta = \frac{(25)^2}{10^{-6}}\times0.2\times10^{-6} = 125°$

Prob-7 For a certain dynamometer ammeter, the mutual inductance M varies with deflection θ (expressed in degrees) as:

$$M = -6\cos(\theta + 30^\circ)\ \mu H$$

Find the deflecting torque produced by a direct current of 50mA corresponding to a deflection of 60°.

Sol: Rate of change of mutual inductance with deflection.

$$\frac{dM}{d\theta} = \frac{d}{d\theta}\left[-6\cos(\theta + 30^\circ)\right] = 6\sin(\theta + 30^\circ)\ \mu H$$

$\frac{dM}{d\theta}$ at a deflection at 60° is

$$\left(\frac{dm}{d\theta}\right)_{\theta=60^\circ} = 6\sin(60^\circ + 30^\circ)\ \mu H$$

$$= 6\times10^{-3}\ H/degree$$

Deflecting torque $T_d = I^2\frac{dm}{d\theta} = (50\times10^{-3})^2 \times 6\times10^{-3}$

$= 15\times10^{-6}\ N/M = 15\ \mu N/M$

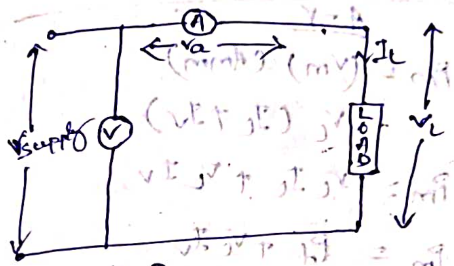
Ch-3 Wattmeters and measurement of power

Power in DC circuits: —

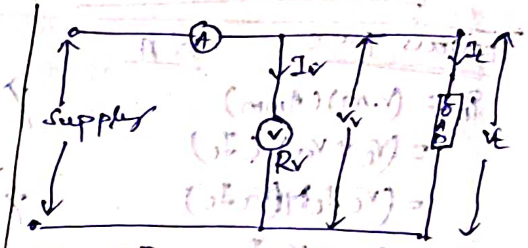
The power taken by a load from a dc supply is given by the product of readings of an ammeter and a voltmeter when connected in the ckt as shown in figure.

Power = VI Watt

V-A method



A-V method



$\Rightarrow P_t = V_L I_L$

$\Rightarrow P_m = (V_M)(A_M)$

$= (V_L + V_a)(I_L)$

$P_t = V_L I_L$

$\Rightarrow P_m = (V_m)(A_{mm})$

$= V_L(I_L + I_V)$

→ Next page

Power in AC ckts: —

The instantaneous power $p = vi$

where, p = Instantaneous power

v = Instantaneous voltage

i = Instantaneous current

→ If both voltage and current waves are sinusoidal and current lagging in phase by an angle φ, then:

$v = V_m \sin\omega t$

$i = I_m \sin(\omega t - \phi)$

So, the instantaneous power,

$p = vi = (V_m\sin\omega t)(I_m\sin(\omega t - \phi))$

$= V_m I_m \sin\omega t \cdot \sin(\omega t - \phi)$

$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$

Note, $\sin A \cdot \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

Average power over a cycle of 2π is,

$$P = \frac{V_m I_m}{2} \times \frac{1}{2\pi} \int_0^{2\pi} [\cos\phi - \cos(2\omega t - \phi)]\, d\omega t$$

$$\boxed{P = \frac{V_m I_m \cos\phi}{2}}$$

As $V_{rms} = \frac{V_m}{\sqrt{2}}$, $I_{rms} = \frac{I_m}{\sqrt{2}}$

$$\boxed{P = V_{rms} I_{rms} \cos\phi}$$

Previous page:

V-A	A-V
$P_m = (V_m)(A_{mm})$ $= (V_L + V_a)(I_L)$ $= (V_L I_L) + (V_a I_L)$	$P_m = (V_m)(A_{mm})$ $= V_L (I_L + I_V)$
	$P_m = V_L I_L + V_L I_V$
$P_m = P_L + V_a I_L$	$P_m = P_L + V_L I_V$
$P_m > P_L$	$P_m > P_L$
e is +ve	e is +ve
$P_m = P_L + V_a I_L$ $= P_L + I_L R_a I_L$ $= P_L + I_L^2 R_a$	$P_m = P_L + V_L I_V$ $= P_L + V_L \cdot \frac{V_L}{R_V}$ $= P_L + \frac{V_L^2}{R_V}$
e is due to Ammeter	e is due to voltmeter
suitable for high R load	suitable for low value R load

The error is due to the instrument which is connected to the load side.

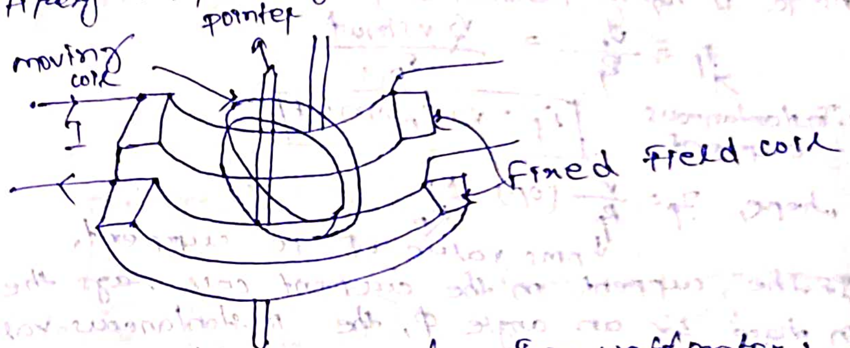
Electrodynamometer wattmeters :-

Construction → The design of electrodynamometer wattmeter is similar to electrodynamometer voltmeter and ammeter.

1) **Fixed coil** :- The fixed coils or field coils are connected in series with the load, so carry the current in the ckt. So the fixed coils form the "current coil" or c.c of wattmeter.

2) **moving coil** :-
The moving coil is connected across the voltage and carries the current proportional to the voltage. So it is called "pressure coil" (or) "voltage coil" or p.c of wattmeter.

→ A high non-inductive resistance is connected in series with the moving coil to limit the current to a small value.
→ Both the fixed coil and moving coils are air cored.
→ The moving coil is mounted on a pivoted spindle and entirely embraced by fixed current coils.



(Electrodynamometer type wattmeter)

3) **Control** :- Spring control is used for the instrument.

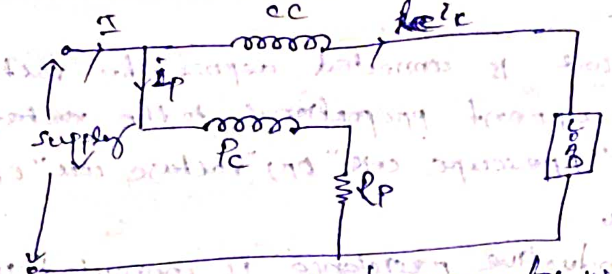
4) **Damping** :- Air friction damping is used as the operating magnetic field is very weak, so eddy current damping can not be used.

5) **Scales and pointers** :- Mirror type scales and knife edge pointers are used to remove the reading errors due to parallax.

Theory of Electrodynamometer wattmeters :-
The instantaneous torque of an electrodynamometer instrument is given by,

$$T_i = i_1 i_2 \frac{dm}{d\theta}$$

where i_1 & i_2 are instantaneous values of current in the two coils.



(Circuit of electrodynamometer wattmeter)

Let, V, I are rms values of voltage and current to be measured.

→ Instantaneous value of voltage across PC is,

$v = \sqrt{2}\, V \sin\omega t$

As PC is highly resistive, current

$i_p = \frac{v}{R_p} = \frac{\sqrt{2}\, V \sin\omega t}{R_p}$

Instantaneous current: $i_p = \sqrt{2}\, I_p \sin\omega t$

where, $I_p = \frac{V}{R_p}$ (or) $\frac{V_p}{R_p}$ → rms value of PC current.

→ If the current in the current coil lags the voltage in phase by an angle ϕ, the instantaneous value of current through CC is:

$i_c = \sqrt{2}\, I_c \sin(\omega t - \phi)$

where I_c = rms value of CC current.

So, instantaneous torque,

$T_i = i_1 i_2 \frac{dm}{d\theta}$

$= i_p i_c \frac{dm}{d\theta}$

$= \sqrt{2}\, I_p \sin\omega t \cdot \sqrt{2}\, I_c \sin(\omega t - \phi) \frac{dm}{d\theta}$

$= 2 I_p I_c \sin\omega t \cdot \sin(\omega t - \phi) \frac{dm}{d\theta}$

$= 2 I_p I_c \left[\frac{1}{2}\left(\cos\phi - \cos(2\omega t - \phi)\right)\right] \frac{dm}{d\theta}$

$T_i = I_p I_c \left[\cos\phi - \cos(2\omega t - \phi)\right] \frac{dm}{d\theta}$

Average deflecting torque,

$T_d = \frac{1}{T} \int_0^T T_i \, d(\omega t)$

$= \frac{1}{2\pi} \int_0^{2\pi} I_p I_c (\cos\phi - \cos(2\omega t - \phi)) \frac{dm}{d\theta}$

$$T_d = I_p I_c \cos\phi \frac{dm}{d\theta} = \frac{V_p}{R_p} I_c \cos\phi \frac{dm}{d\theta}$$

where, $I_p = \frac{V_p}{R_p}$

Controlling torque:

$T_c = K_c \theta$

where, K_c = spring const.

θ = final steady deflection.

At balance position,

$T_c = T_d$

→ $K_c \theta = I_p I_c \cos\phi \frac{dm}{d\theta}$

→ deflection $\theta = \frac{I_p I_c}{K_c} \cos\phi \frac{dm}{d\theta}$

$= \frac{V_p \cdot I_c}{R_p \cdot K_c} \cos\phi \frac{dm}{d\theta}$

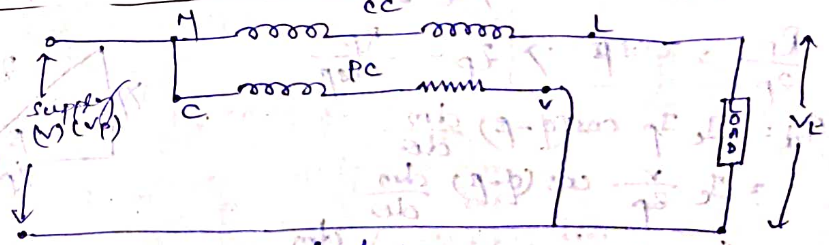
$$\theta = k\, VI \cos\phi \frac{dm}{d\theta}$$

where, $k = \frac{1}{K_c R_p}$, $V_p = V$, $I_c = I$

So, $\theta = \left(k \frac{dm}{d\theta}\right) P$, where, P = power being measured, $P = VI\cos\phi$

Errors in Electrodynamometer wattmeters:–

→ Errors due to pressure coil self inductance:–



C.C → Low R, high L

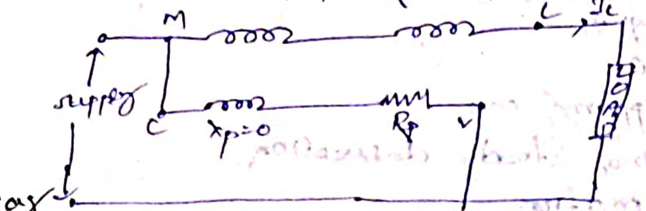
P.C → High R, low L

So drop across CC is less.

$V_L = V_{ph}$

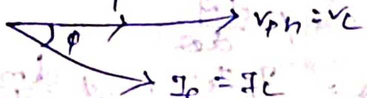
Assumption :-

PC is highly Resistive / X_p (or) $L_p = 0$.
pure 'R'



i) (Case) lag load :-

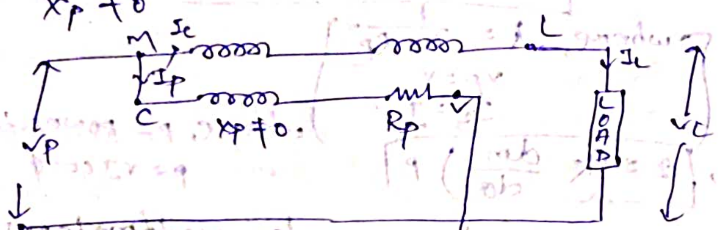
I_p (only R_p) so in phase

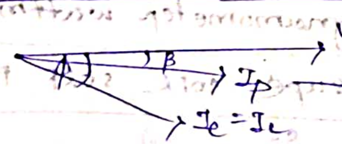


$T_d = I_c\, I_d \cos\phi \dfrac{dm}{d\theta}$

$= \left(I_c \dfrac{V}{R_p} \cos\phi\right) \dfrac{dm}{d\theta}$ → True power.

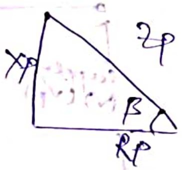
Now, $X_p \neq 0$





As less inductance in P.C so $\beta < \phi$

$\dfrac{R_p}{Z_p} = \cos\beta \Rightarrow Z_p = \dfrac{R_p}{\cos\beta}$



$T_d = I_c\, I_p \cos(\phi - \beta) \dfrac{dm}{d\theta}$

$= I_c \dfrac{V}{Z_p} \cos(\phi - \beta) \dfrac{dm}{d\theta}$

$= \left(I_c \dfrac{V}{R_p} \cos\beta \cos(\phi - \beta)\right) \dfrac{dm}{d\theta}$ → measured power

→ $P_t \propto I_c V \cos\phi$

→ $P_m \propto I_c V \cos\beta \cos(\phi - \beta)$

As $\beta << \phi$
$\cos(\phi - \beta) \uparrow$ $\cos\beta \downarrow$
so $P_m > P_t$

→ $P_m > P_t$

→ ϵ is +ve

→ $P_m \times C.F = P_t$, where CF = correction factor

→ $C.F = \dfrac{P_t}{P_m} < 1$

$C.F = \dfrac{\cos\phi}{\cos\beta \cos(\phi - \beta)} < 1$

→ $\% \epsilon = \dfrac{P_m - P_t}{P_t} \times 100$

$= \left(\dfrac{P_m}{P_t} - 1\right) \times 1000$

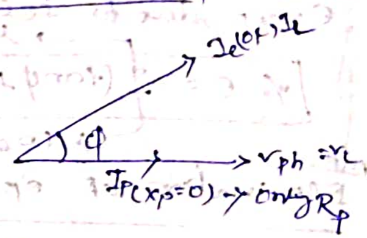
$= \left(\dfrac{1}{C.F} - 1\right) \times 1000$

After solving, $\% = +(\tan\phi \tan\beta \times 100)$

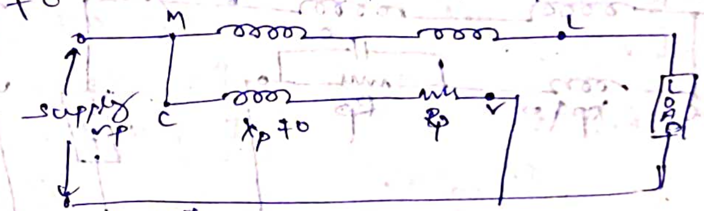
ii) Leading load :-

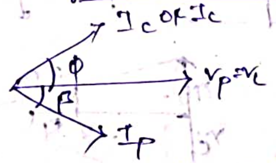
$T_d = I_c\, I_p \cos\phi \dfrac{dm}{d\theta}$

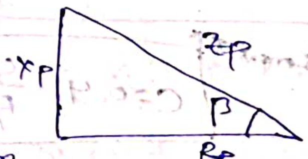
$= \left(I_c \dfrac{V}{R_p} \cos\phi\right) \dfrac{dm}{d\theta}$ → True power.



$X_p \neq 0$







$\dfrac{R_p}{Z_p} = \cos\beta$

$T_d = I_c\, I_p \cos(\phi + \beta) \dfrac{dm}{d\theta}$

$= I_c \cdot \dfrac{V}{Z_p} \cos(\phi + \beta) \dfrac{dm}{d\theta}$

$= \left(I_c \dfrac{V}{R_p} \cos\beta \cos(\phi + \beta)\right) \dfrac{dm}{d\theta}$ → measured power

→ $P_t \propto I_c V \cos\phi$

→ $P_m \propto I_c V \cos\beta \cos(\phi + \beta)$

→ $P_t > P_m$

→ ϵ is -ve

→ $P_m \times C.F = P_t$

→ $C.F = \frac{P_t}{P_m} > 1$

$= \frac{\cos\phi}{\cos\beta\cos(\phi+\beta)} > 1$

→ $\% \epsilon = \frac{P_m - P_t}{P_t} \times 100$

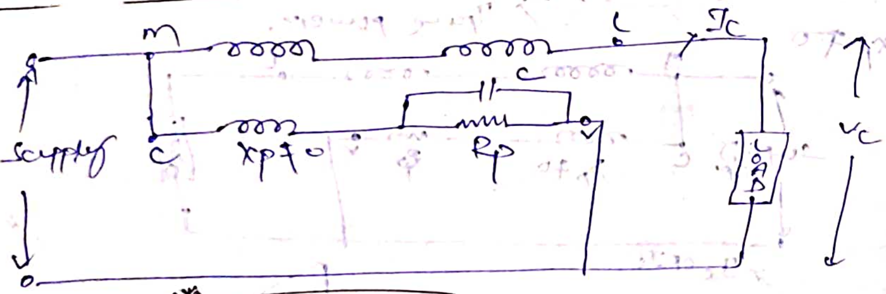
$= \left[\frac{P_m}{P_t} - 1\right] \times 100 = \left[\frac{1}{CF} - 1\right] \times 100$

$\% \epsilon = -(\tan\phi \tan\beta \times 100)$

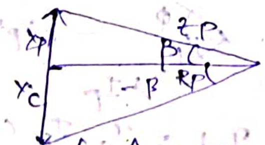
Note
Error due to self inductance of potential coil is
→ lagging load

$\% \epsilon = \pm(\tan\phi \tan\beta) \times 100$
→ leading load.

Imp
Elimination of error due to self inductance :-



Imp***
$C = 0.41 \frac{L_p}{R_p^2}$



→ Error due to self inductance of potential coil can be eliminated by connecting a capacitor. $(C = 0.41 \frac{L_p}{R_p^2})$ in parallel with the resistance of the potential coil as shown in the diagram.

Note →

V.A	A.V	MC-short	LC-short
$P_m = P_t + I_c^2 R_a$	$P_m = P_t + \frac{V_c^2}{R_v}$	$P_m = P_t + I_c^2 R_{cc}$	$P_m = P_t + \frac{V_c^2}{R_{pc}}$

cond^n to have same error
$I_c^2 R_a = \frac{V_c^2}{R_v}$

cond^n to have same error

→ $R_a R_v = \left(\frac{V_c}{I_c}\right)^2$

→ $R_a R_v = (R_L)^2$

→ $R_L = \sqrt{R_a R_v}$

similarly,
$I_c^2 R_{cc} = \frac{V_c^2}{R_{pc}}$

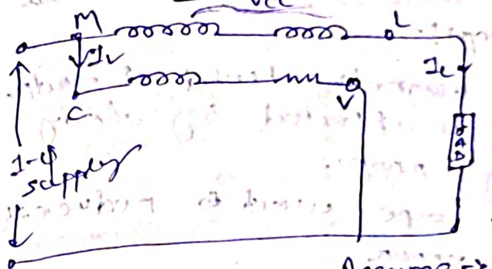
$R_L = \sqrt{R_{cc} R_{pc}}$

(iii) **3-φ power**

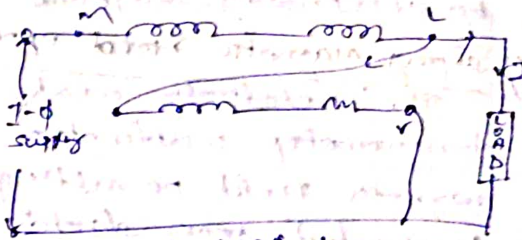
$P_{3-\phi} = 3 \times 1\text{-}\phi$ power
$= 3 V_{ph} I_{ph} \cos\phi$

2) **Errors due to connection**

MC-short



LC-short



Assume → R-load → $\cos\phi = 1$

→ $P_t = V_c I_c$
→ $P_m = P.C \times C.C$
$= V_c + V_{cc} \times I_c$
$= V_c I_c + V_{cc} I_c$
→ $P_m > P_t$
→ ϵ is +ve
→ $P_m = V_c I_c + (I_c) Z_{cc} I_c$
$= V_c I_c + I_c^2 Z_{cc}$
→ ϵ is due to current coil
→ suitable for high R load

→ $P_t = V_c I_c$
$P_m = CC \times P.C$
$= (I_c + I_{pc}) V_c$
$= V_c I_c + V_c I_{pc}$
→ $P_m > P_t$
→ ϵ is +ve
→ $P_m = V_c I_c + V_c \cdot \frac{V_c}{Z_{pc}}$
→ ϵ is due to potential coil
→ suitable for low R load

The error is due to the coil which is connected to the load ckt.

3) **Pressure coil capacitance :-**

→ The pressure coil capacitance is due to interturn capacitance of the series resistance.
→ The effect of capacitance is opposite to the effect of inductance, so wattmeter reads high on lagging power factors of the load.

4) Errors due to mutual inductance effects:–

→ Errors are caused due to mutual inductance betn current and pressure coils of the wattmeters.

→ These errors are quite low at power frequency and increases with the frequency increase.

5) Eddy current errors:–

Eddy currents are induced in the solid metal parts and within the thickness of the conductors by alternating magnetic field of the current coil.

6) stray magnetic field errors:–

The relatively weak magnetic field of electro dynamometer wattmeter is affected by stray magnetic field resulting errors.

→ Laminated iron shields are used to reduce the stray magnetic field error.

7) Errors caused by vibration of moving system:–

If the natural frequency of moving parts like pointer, spring, is in resonance with the torque pulsation the moving systems produce vibration.

→ To avoid the vibration the natural frequency of moving system is very much away from twice the frequency of the system of wattmeter.

8) Temperature errors:–

The change in room temperature changes the resistance of the pressure coil and stiffness of the springs.

To neutralize the effect, the pressure coil ckt is made up of copper and a resistance alloy having a negligible resistance temperature coefficient in the ratio 1:10

ch-4 ~~Energy meters and measurement of energy~~

Induction Type wattmeter:–

Induction type wattmeters are used for the measurement of power only in ac ckts.

Construction:–

→ It consists of two laminated electromagnets wound with conductors known as shunt and series magnets.

shunt magnet:–

→ The upper electromagnet is known as shunt magnet consisting of three limbs.

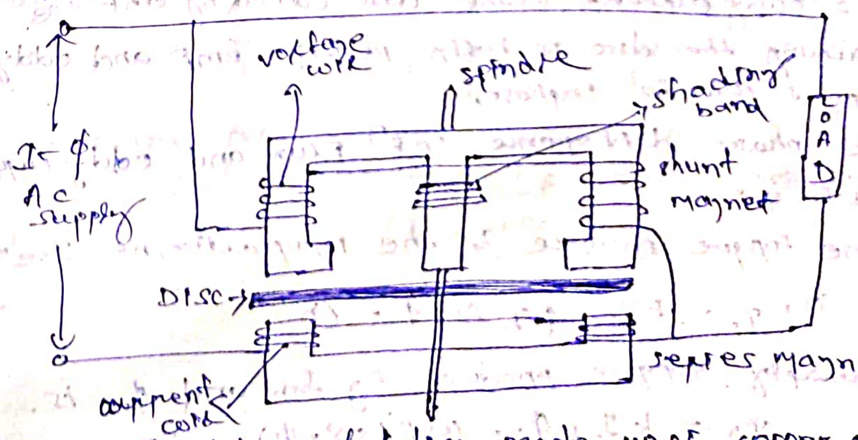
→ The side limbs carry the windings and connected across the load.

→ These windings are excited by the current proportional to the voltage across the load so called voltage coils.

series magnet:–

→ The lower electromagnet is connected in series with load, so known as series magnet.

→ It has two limbs and carries the windings called current coil and excited by the current proportional to load current.



→ A thin light weight disc made up of copper or aluminium is pivoted at the centre in the air gap present betn two electromagnets (shunt and series)

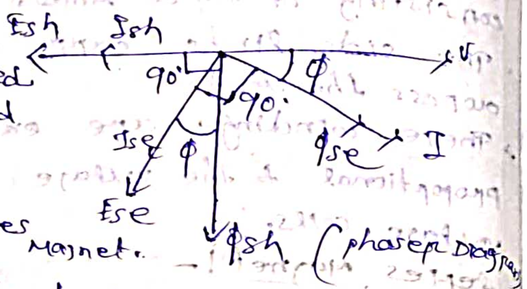
→ It is mounted with the help of spindle and pointer is attached at the top of the spindle.

Working of induction type wattmeters:-

→ A deflecting torque is produced on the disc due to the interaction of fields produced by the eddy current which causes to rotate the disc, than spindle and pointer.

→ To provide controlling torque, spring control method is used.

Torque Equation



Let, V = voltage to be measured

I = current to be measured

ϕ = phase angle betⁿ V & I

ϕ_{se} = Flux produced by series magnet.

ϕ_{sh} = Flux produced by shunt magnet.

E_{sh} = Emf induced in the disc by shunt magnet flux

E_{se} = Emf induced in the disc by series magnet flux

I_{sh} = Eddy current in the disc caused by emf E_{sh}

I_{se} = Eddy current in the disc caused by emf E_{se}

→ Assuming the disc is fully resistive emf and eddy current will be inphase.

→ The phase difference betⁿ flux and eddy current will be 90°.

The torque produce by the interaction of I_{se} and ϕ_{sh} is; $T_1 = K I_{se} \phi_{sh} \cos\phi$ — (1)

Similarly torque produce by I_{sh} and ϕ_{se} is.

$T_2 = K I_{sh} \phi_{se} \cos(180° - \phi)$ — (2)

So the resultant torque.

$T_d = T_1 - T_2$

$= K I_{se} \phi_{sh} \cos\phi - K I_{sh} \phi_{se} \cos(180° - \phi)$

$T_d = K[I_{se} \phi_{sh} \cos\phi + I_{sh} \phi_{se} \cos\phi]$

As, $I_{se} \propto I$

$\phi_{sh} \propto V \propto I_{sh}$

$T_d = K[K_1 VI \cos\phi + K_2 VI \cos\phi]$

$T_d \propto KVI \cos\phi$, $T \propto P$ → power

Advantages:-

The scale is uniform

→ They provide good damping

→ No effect of stray magnetic field.

Disadvantages:-

→ Can be used only for ac power measurement.

→ Low accuracy due to heavy moving system.

→ Error due to change in temperature

→ Power consumption is more.

Difference betⁿ electrodynamometer & Induction type watt meter

Electrodynamometer type	Induction type
→ Can be used for both ac and dc power	→ used only for ac power
→ single pressure coil	→ Both current and pressure coils are superfed into two parts.
→ pressure coil is the moving part to which pointer is attached.	→ Aluminium disc the moving part to which pointer is attached.

ch-8 **Energy meters and measurement of energy** Dt-28

→ Energy is the total power delivered or consumed over a time interval.

Energy = power × time

$$W = \int_0^t vi\,dt$$

where, v = volt, i = Ampere, t = sec

→ so unit of energy is joule or watt second.

→ kilo watt hour - energy consumed when power is delivered at an average rate of 1000 watt for one hour.

Meter constant:-

Meter constant is defined as the no. of revolution made per kilowatt-hour (kwh)

Braking:-

→ The speed of the moving system is controlled by braking system.

→ Braking system is made up of permanent magnet (breaking magnet) placed near the moving system which produces eddy currents in some part of the moving system.

→ The eddy currents produces a braking (retarding) torque which is proportional to the speed of moving system.

→ As eddy currents are produced in the aluminium disc, the moving system (disc) cuts through the field of permanent magnet.

Emf generated in the disc,

$$e = k_1 \phi n$$

where, ϕ = flux of permanent magnet (Braking magnet)

n = speed of rotation

k_1 = constant.

When r = resistance of eddy current path.

eddy current $i = \frac{e}{r} = \frac{k_1 \phi n}{r}$ —(1)

The braking Torque (T_B) ∝ flux of magnet

∝ Eddy current

∝ effecting radious R' of the Al. dis

so, Braking Torque,

$$T_B = k_2 \phi i R$$

$$T_B = \frac{k_1 k_2 \phi^2 n R}{r} \quad [\text{putting eq}^n (1)]$$

$$T_B = \frac{k_3 \phi^2 n R}{r} \quad [\text{where } k_1 k_2 = k_3]$$

→ If the radious of the disc R = constant.

Then, $T_B = \frac{k_4 \phi^2 n}{r}$ where, $k_3 R = k_4$

→ The moving system catains a steady speed when the driving torque is equal to the braking torque.

Braking torque at steady speed N is,

$$T_B = \frac{k_4 \phi^2 N}{r} = kN$$

where, $k = \frac{k_4 \phi^2}{r}$

→ At steady speed Braking Torque = Deflecting Torque

$$\Rightarrow T_B = T_d$$

$$\Rightarrow kN = T_d \Rightarrow \boxed{N \propto T_d}$$

Theory of Induction Type Energy meter:-

→ In induction type energy meter, two fluxs are produced by current flowing in the windings of current coil and potential coil of series magnet and shunt magnet respectively.

→ For two fluxes two eddy currents are produced so two torques are produced.

1) First flux interacting with eddy current produced by the second flux

2) second flux interacting with eddy current produced by the first flux

Let ϕ_1 and ϕ_2 be the instantaneous values of two fluxes having a phase difference of β.

$\phi_1 = \phi_{m1} \sin\omega t$; $\phi_2 = \phi_{m2} \sin(\omega t - \beta)$

where, ϕ_{m1}, ϕ_{m2} = max. value of fluxes.

$\left.\begin{array}{l}\phi_1 = \phi_{m1}/\sqrt{2} \\ \phi_2 = \phi_{m2}/\sqrt{2}\end{array}\right\}$ rms value of fluxes.

→ Flux ϕ_1 produces an emf in the disc by transformer action

$$e_1 \propto \left[-\frac{d(\phi_1)}{dt}\right] \propto \left[-\frac{d}{dt}(\phi_{m1}\sin\omega t)\right]$$

$$\propto [-\omega \phi_{m1} \cos\omega t] \propto [-f \phi_{m1} \cos\omega t]$$

(lagging)

e_1 lags the flux by 90° as ϕ in $\sin\omega t$ & e in $\cos\omega t$ phase

Let, E_1 = rms value of emf e_1

$$E_1 \propto f \frac{\phi_{m1}}{\sqrt{2}} \propto f \phi_1$$

Then the eddy current $\boxed{I_1 = \frac{E_1}{Z} \propto \frac{f \phi_1}{Z}}$

where f = frequency

ϕ_1 = rms value of flux

Z = Impedance of eddy current path.

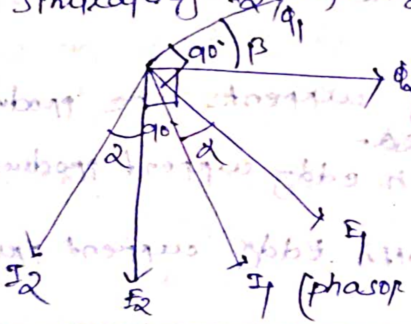
→ I_1 lags the voltage E_1 by angle α.

Similarly $E_2 \propto f \frac{\phi_{m2}}{\sqrt{2}} \propto f \phi_2$

Then eddy current

$$\boxed{I_2 = \frac{E_2}{Z} \propto \frac{f \phi_2}{Z}}$$

Similarly I_2 lags E_2 by angle α



(phasor diagram of induction type energy meter)

Angle bet^n I_1 & ϕ_2 is $(90° - \beta + \alpha)$

Angle bet^n I_2 and ϕ_1 is $(90° + \beta + \alpha)$

→ Now average torque produced by the interaction of I_1 and ϕ_2 is

$$T_{d1} \propto I_1 \phi_2 \cos(90° - \beta + \alpha)$$

$$\propto f \frac{\phi_1}{Z} \phi_2 \cos(90° - \beta + \alpha)$$

→ Similarly average torque produced by the interaction of I_2 and ϕ_1 is

$$T_{d2} \propto I_2 \phi_1 \cos(90° + \beta + \alpha)$$

$$\propto f \frac{\phi_2}{Z} \phi_1 \cos(90° + \beta + \alpha)$$

Total deflecting torque

$$T_d = T_{d1} - T_{d2}$$

$$\propto \frac{\phi_2 \phi_1}{Z} f \cos(90 - \beta + \alpha) - \frac{\phi_2 \phi_1}{Z} f \cos(90° + \beta + \alpha)$$

$$\propto \frac{\phi_2 \phi_1}{Z} f \left[\sin(\beta - \alpha) + \sin(\beta + \alpha)\right]$$

Note, $\cos(90 - \beta + \alpha) = \cos[90 - (\beta - \alpha)]$
$= \cos(90° - \theta)$
$= \sin\theta$

$\cos(90 + \theta) = -\sin\theta$

$$\propto \frac{\phi_2 \phi_1}{Z} f \left[\sin(\beta - \alpha) + \sin(\beta + \alpha)\right]$$

$\sin(a+b) + \sin(a-b) = 2\sin A \cdot \cos B$

$\sin(\beta + \alpha) + \sin(\beta - \alpha) = 2\sin\beta \cdot \cos\alpha$

Deflecting torque, $T_d \propto \frac{\phi_1 \phi_2}{Z} f (2\sin\beta \cdot \cos\alpha)$

$$\boxed{T_d \propto \frac{\phi_1 \phi_2}{Z} f \sin\beta \cos\alpha}$$

where, β = angle bet^n ϕ_1 and ϕ_2

α = angle bet^n voltage and emf current (E & I)

For max. torque,

'α' should be nearly equal to '0',

so $\cos 0° = 1$

and 'β' should be nearly equal to 90°,

so, $\sin 90° = 1$

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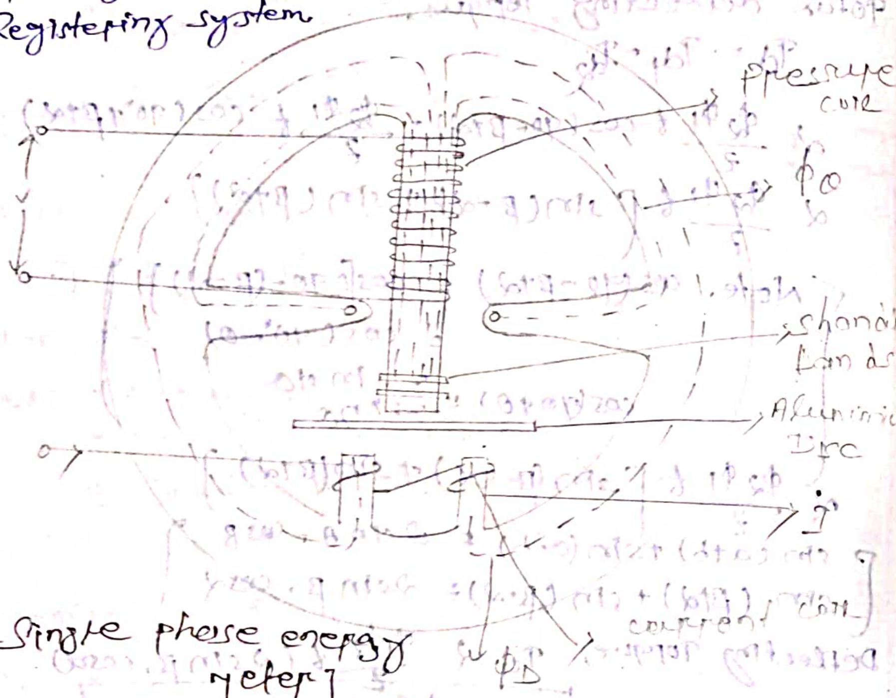
Energy meters and measurement of energy

Single phase Induction Type meters:—

Construction:—

There are four main parts of the operating mechanism:—

1) Driving system
2) Moving system
3) Braking system
4) Registering system



[Single phase energy meter]

1) Driving system

→ The driving system of the meter consists of two electromagnets made up of silicon, steel laminations.

→ One of the electromagnet is excited by load current known as current coil carries current proportional to the supply voltage known as pressure coil.

→ Current electromagnet is also known as series magnet and pressure electro-magnet is also known as shunt magnet.

→ Copper shading bands are placed on the central limb of shunt magnet whose function is to bring the flux produced by the shunt magnet exactly in quadrature with the applied voltage.

2) Moving system:—

It consists of an aluminium disc mounted on the light alloy shaft which is positioned in the air gap bet^n series and shunt magnets.

3) Braking system:—

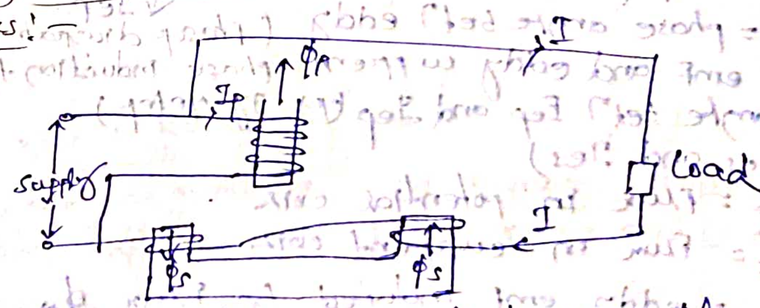
A permanent magnet placed near the edge of aluminium disc forms the braking system.

→ The position of the permanent magnet is adjustable to adjust the braking torque.

4) Registering (Counting) System:—

→ The function of a registering or counting system is to record continuously a number which is proportional to the revolution made by moving system.

Theory and operation of single phase energy meters:—



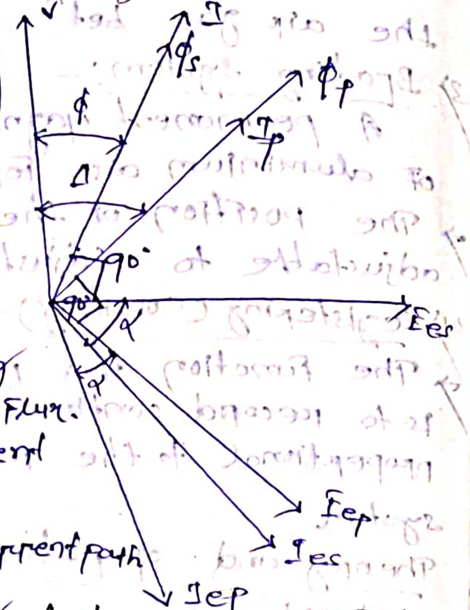
→ The supply voltage is applied across the pressure coil which is highly inductive, so the current I_P through the pressure coil is proportional to the supply voltage and lags it by a few degree less than 90° (Δ).

→ I_p produces ϕ_p and induces eddy emf ~~Esp~~ E_{ep} and current I_{ep} (Eddy current in potential coil) which lags E_{ep} by angle 'α'.

→ Similarly load current 'I' flows through current coil and produces flux ϕ_s.

→ ϕ_s proportional to load current and inphase with I.

→ ϕ_s induces eddy emf E_{es} and induces current I_{es} which lags E_{es} by an angle 'α'.



(phasor diagram of single phase induction type energy meter)

Note
* angle bet^n ϕ_p & ϕ_s = $\Delta - \phi$
* angle bet^n E & I = α

Let, V = applied voltage

I = Load current

ϕ = phase angle bet^n V & I

Δ = phase angle bet^n supply voltage and pressure coil flux.

I_p = pressure coil current

f = Frequency

Z = Impedance of eddy current path

α = phase angle bet^n eddy emf and eddy current (angle bet^n E_{ep} and I_{ep} (or) E_{es} and I_{es})

ϕ_p = Flux in potential coil

ϕ_s = Flux in current coil

E_{ep} = eddy emf induced by flux ϕ_p

I_{ep} = eddy ~~emf induced~~ current due to flux ϕ_p

E_{es} = eddy emf induced by flux ϕ_s

I_{es} = eddy current due to flux ϕ_s

Net deflecting torque

$$T_d \propto \phi_1 \phi_2 \frac{f}{Z} \sin\beta \cos\alpha$$

where, $\phi_1 = \phi_p$

$\phi_2 = \phi_s$

β = angle bet^n ϕ_p and ϕ_s

α = angle bet^n emf and current

So, $T_d = K_1 \phi_p \phi_s \frac{f}{Z} \sin(\Delta - \phi) \cos\alpha$

But, $\phi_p \propto V$, $\phi_s \propto I$

So, $T_d = K_2 V I \frac{f}{Z} \sin(\Delta - \phi) \cos\alpha$

If f, Z and α are constant,

$$T_d = K_3 V I \sin(\Delta - \phi)$$

→ If, N = steady speed,

braking torque, $T_B = K_4 N$

At steady speed; the driving torque must equal to breaking torque

$$T_B = T_d$$

$\Rightarrow K_4 N = K_3 V I \sin(\Delta - \phi)$

If $\Delta = 90°$

$\Rightarrow K_4 N = K_3 V I \sin(90° - \phi)$

$\Rightarrow K_4 N = K_3 V I \cos\phi$

$\Rightarrow$ $\boxed{N = K V I \cos\phi}$

$\Rightarrow$ $\boxed{N = K \times \text{power}}$

So, for registering the energy, speed of rotation 'N' proportional to power and Δ must be equal to 90°

Total no of revolution $= \int N dt$

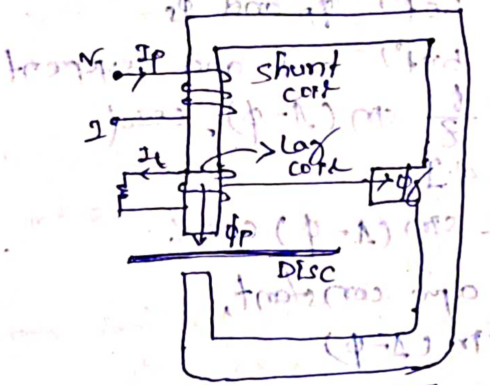
$= K \int (V I \cos\phi) dt$

$= K \int (\text{power}) dt$

$\boxed{N = K \times \text{energy}}$

Lag adjustment Devices :-

→ The meter will register true energy, when the angle Δ i.e angle betⁿ supply voltage (V) and pressure coil flux (ϕ_p) is equal to 90°.

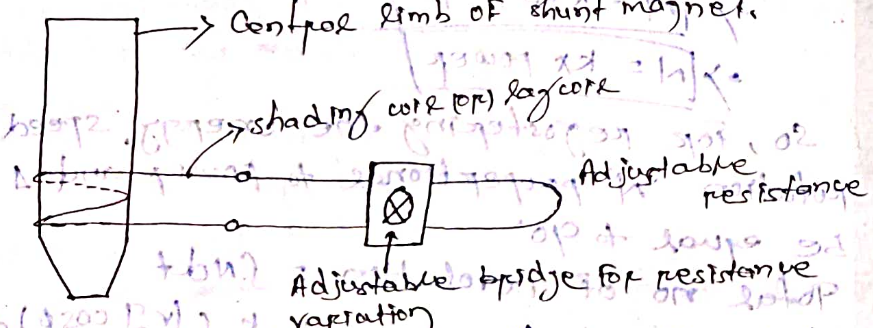


→ So a magnetic shunt circuit is introduced which by passes some portion of shunt magnet flux in the gap (ϕ_g) to make Δ exactly 90° (quadrature)

→ The required mmf is obtained from a lag coil which is located on the central limb of shunt magnet close to the disc gap.

→ For adjusting the mmf of lag coil, following arrangements are done.

1.) Adjustable resistance :-



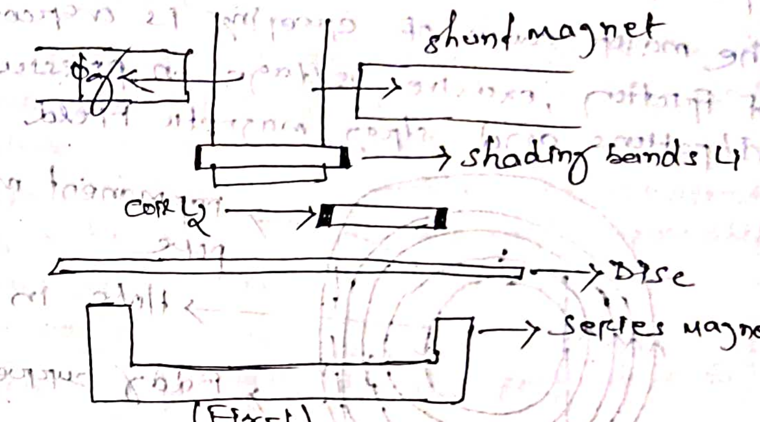
→ The resistance of the circuit is altered to adjust the lag angle of flux ϕ_p.

→ When resistance is increased, the current decreases, MMF in the lag coil decreases and value of lag angle θ is decreased.

→ Similarly when resistance is decreased, the current and mmf of lag coil is increased, so value of lag angle θ is increased.

→ The value of lag angle is adjusted to make Δ=90°.

2) Shading bands :-



(Fig-1)

In this case copper shading bands L1 are placed around the central limb of shunt magnet instead of lag coil with adjustable resistance.

→ When shading bands are moved upward more fluxes are linked which increases the induced emf, current and mmf, so lag angle θ increases.

→ Similarly, when shading bands are moved downward, flux linkage decreases, so emf current, mmf also decreases and the lag angle θ decreases.

→ The value of lag angle is adjusted to make Δ=90°.

Light load (or) Friction compensation :-

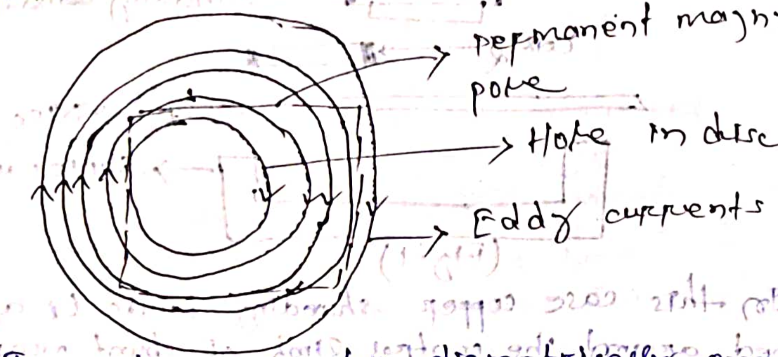
→ Friction errors are more in case of light loads, so a small torque is required which acts in the direction of rotation and which is nearly equal to friction torque.

→ This torque is obtained by a small shading loop situated betn the central limb of shunt magnet and the disc which is coil (b) (Fig-1)

Creep!—

The continuous rotation of meter when no current flowing through the current coil but pressure coil is energised is called creeping.

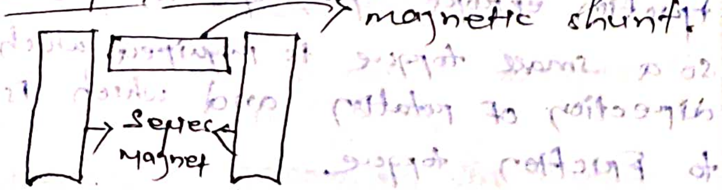
The major cause of creeping is overcompensation of friction, excessive voltage on pressure coil, vibrations and stray magnetic field



To avoid creeping, two diametrically opposite holes are drilled on the disc. When the hole is under the edge of pole, the circular eddy current paths in the disc will be distorted and the creep will be stopped.

In some other cases, a small piece of iron is attached to the edge of the disc, so the force of attraction exerted by the brake magnet on the iron piece is sufficient to prevent creeping of disc

Overload compensation!—



→ Due to the rotation of disc continuously in the field of series magnet under load condition a dynamically emf is induced in the disc.

→ This dynamically induced emf creates braking torque which is proportional to the square of load current.

→ So when the load increases the self braking torque increases.

In order to minimize the selfbraking action.

a) The full load speed of the disc is kept as low as possible i.e nearly 40 rpm.

b) a overload compensating device i.e a magnetic shunt is used which saturates and permeability decreases at overload.

Voltage compensation!—

voltage variation causes error due to following reasons.

i) the relationship betn shunt magnet flux and supply voltage is not linear

ii) The self braking dueto dynamically induced emf in the disc by shunt magnet.

→ To minimize the voltage variation errors saturable magnetic shunt is used

Temperature compensation!—

When temperature increases:-

→ potential coil flux ϕ_p decreases and lag angle also decreases.

→ Torque by shading bands decreases.

→ Resistance of eddy current path increases.

→ Lag angle 'α' of eddy current path decreases.

→ The increase in temperature causes the meter to run fast and register high.

i) Temperature compensation is done by using a temperature shunt on the brake magnet which is made up of special magnetic material like (mutemp) in which permeability decreases when ~~teper~~ temperature increases.